Passive Model Reduction and Switching for Fast Soft Object Simulation with Intermittent Contacts

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Abstract—We propose a novel fast simulation framework for soft objects/robots with intermittent contacts, whose contact areas/locations can be varying. We first perform a balanced model reduction of the full-order FEM (finite element method) model for each contact mode with the contact force as the input and the shape of the object/robot as the output. We then devise the strategy of passive model reduction and passive model switching of these reduced-order models (each with its contact mode) utilizing the techniques of our recently-proposed passive mid-point integration (for the passivity of each reduced-order model) and simultaneous diagonalization (for the passivity of model reduction and model switching). The efficacy of the theory is then demonstrated with simulation and experimental results.

I. INTRODUCTION

Soft robots recently have received significant interests in the robotics community and from the general public, since they would enable soft/compliant contacts with external environments and, in particular, with humans, while also being inherently safe to interact with. Being soft itself also opens up a new avenue of research as compared to the conventional “rigid” robots, with its possible applications likely not yet fully-unfolded/fathomed. See [1], [2], [3] for some recent developments in the field of soft robotics.

In this paper, we propose a novel fast simulation framework for soft objects (and possibly robots) with intermittent contacts. Here, by intermittent contacts, we mean that contacts are applied to different contact areas/locations, yet, sequentially with some (almost) zero contact forcing interval in between them, whose duration is larger than the dynamics of the soft objects. Each contact area may be separated from each other, have some overlaps with one another, or even be nested with each other. We allow the contact actuation (e.g., tendon force for soft robots) to be continuously applied though (with its effect contained in all some certain input matrices - see Sec. II). Although this assumption of intermittent contact is certainly limited to incorporate general contact behaviors, we yet still believe that it still would enable us to attain many practically important application scenarios in soft robotics or for soft object robotic manipulation - see Sec. IV. Fully exploiting this intermittency of contacts also allows us to significantly speed up the simulation (again, see Sec. IV), which we also believe could be fairly useful for some applications.

This problem of soft robots (or objects) simulation is challenging. One of the fundamental challenges is that the model of soft objects/robots, typically formulated by a FEM (finite element method [4], [5]), is of a very large number of nodes, making it difficult to simulate practically fast enough. This fast simulation, however, is necessary not only for conventional development processes of soft robotics or soft object robotic manipulation (e.g., quick verification of early design, design and tuning of nominal control laws, etc.) but also for recent directions of applying data-driven/learning techniques for those [6], [7]. This challenge of large-dimensional dynamics is further exacerbated in the presence of contacts with external environments/objects, particularly, when the locations of those contact force applying are varying, since such otherwise-working model reduction techniques as PCA (principal component analysis [8]), POD (proper orthogonal diagonalization [9], [10]) or balanced model reduction (BMR, [11], [12]) cannot be directly applied. In fact, to our knowledge, how to combine those model reduction techniques for the case of contacts with changing locations is still an open problem in the fields of robot simulation and structural mechanics.

To address these challenges, in this paper, we adopt a framework of local model reduction for each intermittent contact with the model switching according to the contact areas/locations. More precisely, for each contact mode, we perform BMR of the full-order FEM model with the contact force as the input and the shape of the soft object as the output. Here, we choose this (analytical) BMR rather than those (data-driven) PCA/POD methods, since, for some scenarios, where it is difficult to produce/compute the force of all the contact-related nodes for “representative” contact forcings, we believe this analytical method would be preferred. We then switch among these reduced-order models (each with its contact mode) to address the changing locations of the intermittent contacts. This switching, however, can trigger instability just as switching of stable systems can be unstable [13], [14]. To ensure the stability of this switching, in this paper, we render this switching among different reduced-order models (with different contact mode) to be passive. For this, we utilize our recently-proposed PMI (passive mid-point integration [15]) to enforce the passivity of each reduced-order model and also fuse the BMR with simultaneous diagonalization to enforce the passivity of the model reduction and model switching. The devised theory is then experimentally
demonstrated.

Some (very) recent FEM-based results on this soft object or soft object robotic manipulation are [16], [17], all of which, however, is limited to the contact point to be fixed throughout the operations due to the reasons as stated above. The more relevant result to this paper is [18], where the authors utilize POD-based model reduction with the “global” fitting of all the contact forces. In contrast to this, in this paper, we adopt “local” BMR with each contact mode and switch among the reduced-order models, which we believe can be more accurate just as so with (local) linear and (global) nonlinear controls, although the more rigorous comparison is necessary here [19]. Also, to our best knowledge, this paper is the first to analyze the performance of real-time simulation of a soft object based on experimental data of the actual system - see Sec. IV. BMR has also been frequently used in the field of structural mechanics, yet, typically with no consideration of contact force with location changes [12], [20], [21]. Another interesting line of research is to simulate soft objects/robots with EKC (elastic kinematic chain) or cosserat rod, both being essentially 1-dimensional simulation [22], [23], [24], thus, cannot provide as complex behavior and shape change as the FEM approach can.

The rest of the paper is organized as follows. Sec. II contains some preliminary materials, including the problem set with the FEM model and a brief review of PMI. Sec. III contains the main results of this paper, that is, passive model reduction (for each contact mode) and passive switching among those models, based on the techniques of PMI and simultaneous diagonalization. Sec. IV presents experimental demonstration of our proposed framework. Some concluding remarks are in Sec. V.

II. PRELIMINARY

A. FEM Modeling with Input Matrix Switching

To model soft objects (or robots), we adopt the FEM, which separates the objects into multiple finite elements, each in turn with multiple nodes. Define the position of the $i$-th node by $p_i := [p_i^x, p_i^y, p_i^z] \in \mathbb{R}^3$. Then, the configuration of the soft object can be written by

$$x := [p_1; p_2; \ldots; p_N] \in \mathbb{R}^N$$

where $N \in \mathcal{N}$ is the total number of the nodes and $n \in \mathcal{N}$ is the total dimension of the soft object with $n := 3N$. The dynamics of the soft object is then described by the following large-size FEM model:

$$\Sigma_x: M \ddot{x} + D \dot{x} + Kx = B_i u_i$$

(1)

where $M, D, K \in \mathbb{R}^{n \times n}$ are the inertia, damping, and stiffness matrices, all symmetric and positive definite; and $u_i \in \mathbb{R}^p$ is the contact forcing term, which may also include control actuation; and $B_i \in \mathbb{R}^{n \times p}$ is the input matrix specifying the subset of nodes on which $u_i$ is applied.

In (1), we adopt the notation of $B_i u_i$, since, in many scenarios of robotics, such contact force is not applied arbitrarily on all the nodes but rather on specific subset nodes at each given time. For this, define the set of contact modes $C = \{1, 2, \ldots, N\}$, where $N_c$ is the cardinality of the contact modes and each $i \in C$ is associated with its input matrix $B_i$. We also assume that all the input matrices $B_i$ contain the control actuation. In this paper, we assume the contacts be intermittent, i.e., $B_i$ switches to $B_j$ with $u_i = 0$ (except control actuation) for an interval longer than the $\Sigma_x$-dynamics (1). Even if it is certainly limited, we still believe this intermittent assumption is still relevant to many scenarios of robotic manipulations, for which our full leverage of the contacts being intermittent can substantially speed up the simulation - see Sec. IV. In contrast, the control actuation (within $u_i$) needs not to be intermittent, as it is contained by all the input matrices $B_i$, thus, their subspace is shared by all $B_i$ due to the linearity of the FEM dynamics (1).

Here, we also assume that the deformation of the soft object is not so large (e.g., plastic snap connector/case, etc.) with $M, D, K$ in (1) all being constant as well, although the presented results can still be adequately applied to soft objects with larger deformations with some reasonable accuracy - see Sec. IV. These constant matrices $M, D, K$ capture the material properties, object shape, boundary condition, etc. Here, note also that $M, D$ are often assumed to be constant in other works (e.g., [18]). How to extend the result presented in this paper to the case of non-constant $K$ (e.g., $K(x)$) is a topic for future research, for which the idea of passive switching in Sec. III could be extended to the passive (i.e., stable) switching of stiffness $K_m$ approximating $K(x)$.

The dimension $n$ of the soft object (or robot) FEM model (1) is typically fairly large (e.g., tens of thousands in Sec. IV). For the real-time simulation of the soft object as aimed in this paper, it is then necessary to reduce the order of the original large-size FEM model (1) to those, which is small enough for computation speed, yet, at the same time, still large enough for simulation accuracy. Such reduced dynamics should also properly incorporate the forcing mode (i.e., input matrix $B_i$), since, for instance, the dominant mode shapes are in general dictated by $B_i$ (i.e., location of the external forcing/loading). To address this, in this paper, we will utilize the technique of BMR, which explicitly takes into account $B_i$ to obtain the set of (most controllable) reduced modes.

Further, since this mode of loading $B_i$ generally changes during the operations, we will switch among those reduced-order models depending on this mode of external forces/loading $B_i$. However, the processes of the model reduction and the switching among the reduced-order models are not necessarily energetically-consistent (i.e., passive). For instance, similar to the case of switched systems, where, even if each system is stable, their switching may trigger instability, the switching process itself may induce energy increase, thereby, possibly trigger the instability of the whole simulation, even if each reduced system is energy-preserving (or passive with power in/out-flow incorporated - see Sec. II-B). To circumvent this issue of switching-induced instability, here, we will enforce the passivity [19], [25] of the processes of model reduction and switching among the reduced-order
models. A prerequisite for this is to enforce the passivity of each reduced-order system, and, for that, we adopt our recently-proposed PMI (passive mid-point integration [15], [26]) for integrating the reduced-order models of the FEM model (1), each with different $B_i$, as briefly reviewed in Sec. II-B.

B. Passive Midpoint Integration [15]

To enforce passivity of each reduced-order model, in this paper, we utilize our recently-proposed technique of PMI (passive mid-point integration [15], [26]). Since the reduced-order models are to be linear as so for the original FEM (passive mid-point integration [15], [26]). Since the reduced-order models are to be linear as so for the original FEM model reduction techniques.

Here, note that this $z_i$ contains from the element of $\bar{z}_i$ with the largest Hankel singular value to the element of $\bar{z}_i$ with the largest Hankel singular value to...

For the application of the BMR, we first rewrite the FEM dynamics into the form, which can be converted into the standard first-order state-space form:

$$\begin{bmatrix} \dot{V} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} -D & -K \\ I & 0 \end{bmatrix} \begin{bmatrix} \dot{V} \\ x \end{bmatrix} + \begin{bmatrix} B_i \end{bmatrix} u_i$$

Here, note that, even if converted, this form is different from a standard state-space representation, since there should be kinematic consistency among its state variables $x$, i.e., since $X(1:n) = \dot{x}$ and $X(n+1:2n) = x$, we should enforce $X(1:n) = \dot{X}(n+1:2n)$. Standard BMR, in general, does not discriminate state variables according to their kinematic relations. Thus, to attain the BMR while enforcing the second-order structure of the FEM model dynamics (1), here, we adopt the second-order BMR result of [28].

More precisely, we first define $u_i$ as the system input and $x = [0_{n \times n}, I_{n \times n}] X$ as the output. We choose this output $x$, since what we want to focus here is the shape of the soft object. We can then compute the controllability and observability Gramians $W_{c_i}$:

$$W_{c_i} = \begin{bmatrix} W_{c_i,uu} & W_{c_i,uv} \\ W_{c_i,vu} & W_{c_i,xx} \end{bmatrix}$$

where all block matrices are $(n \times n)$-dimensional matrices. We then choose only those relevant to the configuration $x$ (i.e., $W_{c_i,xx} \in \mathbb{R}^{n \times n}$) and perform the balanced realization as if this $x \in \mathbb{R}^n$ is the only the state of the system, i.e.,

$$x =: T_i \bar{z}_i, \quad \bar{T}_i := R_i^T U_i \Lambda_i^{\frac{1}{2}} \in \mathbb{R}^{n \times n}$$

where $R_i^T R_i := W_{c_i,xx}$, $U_i \Lambda_i^{\frac{1}{2}} U_i^T := R_i W_{o,xx} R_i^T$ with $U_i^T U_i = U_i U_i^T = I$. Here, $\Lambda_i \in \mathbb{R}^{n \times n}$ contains the Hankel singular values with the controllability and observability Gramians of $\bar{z}_i \in \mathbb{R}^n$ “balanced” with $W_{c_i,\bar{z}_i \bar{z}_i} = W_{o,\bar{z}_i \bar{z}_i} = \Lambda_i$. See [11].

The BMR is then obtained by choosing

$$z_i := \begin{bmatrix} \bar{z}_i^1, \bar{z}_i^2, \ldots, \bar{z}_i^m \end{bmatrix} = P \bar{z}_i \in \mathbb{R}^m$$

from $\bar{z}_i \in \mathbb{R}^n$, where $P = [I_{m \times m}, 0_{m \times (n-m)}] \in \mathbb{R}^{m \times n}$ is the selection matrix. Here, note that this $z_i$ contains from the element of $\bar{z}_i$ with the largest Hankel singular value to...
that with the \( m \)-th largest value. We choose the order of the reduced-order model (i.e., \( m \)) to be large enough to retain the accuracy of the model reduction, while also being small enough for simulation speed. For this paper, we also set this \( m \) to be the same across different \( B_i \). This is done only for simplicity - how to extend it to different \( m_i \) for different \( B_i \) is rather straightforward. Now, rewrite \( z_i \in \mathbb{R}^m \) s.t.,

\[
z_i = Pz_i = PT_{i}^{-1}x \quad (7)
\]

We can then obtain the reduced dynamics of \( z_i \in \mathbb{R}^m \) as follows: with \( z_i^{m+1} = z_i^{m+2} = \ldots = z_i^n = 0 \), \( z_i = PT_{i}z_i \in \mathbb{R}^n \) and (7),

\[
\Sigma_i^z : \ M_i \ddot{z}_i + D_i \dot{z}_i + K_iz_i = G_iu_i \quad (8)
\]

where \( M_i, D_i, K_i \in \mathbb{R}^{m \times m} \) are the transformed positive definite (or semi-definite) and symmetric inertia, damping and stiffness matrices, and \( G_i \in \mathbb{R}^{m \times p} \) is the transformed input matrix.

Using the PMI formulation (2)-(3), this reduced dynamics \( \Sigma_i^z \) in (8) can be simulated while enforcing discrete-time passivity \( (4) \). However, for the case of intermittent contacts, if we capture all those contact forcings with one input matrix \( B_i \), the model reduction maybe not so effective with the order of \( \Sigma_i^z \) still too large for fast simulation. For such cases, it would be more efficient to have multiple reduced-order models \( \Sigma_i^z \) for each \( B_i \) (i.e., \( \mathcal{C} \)) and switch among these models \( \Sigma_i^z \) depending on \( B_i \). This switching, however, may induce overall simulation instability, just as switching of stable systems can be unstable. To ensure the stability of this model switching, in this paper, we aim to enforce the passivity of this model switching. More precisely, we aim to render the energetics of this switching as shown in Fig. 1: from \( \Sigma_i^z \), the full state \( x_i = T_iPT_{i}^Tz_i \) is reconstructed (i.e., \( \Sigma_i^z \rightarrow \Sigma_i^z \)) and \( B_i \) is switched to \( B_j \) (i.e., \( \Sigma_i^z \rightarrow \Sigma_j^z \)) and reduced again into \( \Sigma_j^z \) (i.e., \( \Sigma_j^z \rightarrow \Sigma_j^z \)) with total energy \( V_k \) not increasing. In this case, the process of reconstructing the full state preserves energy because it is only adding the element with zero in the existing reduced state. Also, changing the input matrix does not affect the state, so the energy is also preserved. Therefore, a prerequisite for enforcing passivity of model switching is that the energy of the reduced-order model (i.e., \( \Sigma_j^z \)) should be no more than that of the full-order model (i.e., \( \Sigma_i^z \)). Rather surprisingly, this is not generally true for the BMR (i.e., \( \Sigma_j^z \rightarrow \Sigma_j^z \)) as shown in the following Prop. 1.

**Proposition 1** The balanced model reduction process (i.e., \( \Sigma_j^z \rightarrow \Sigma_j^z \)) may not be passive, i.e., the total energy \( V_k \) of the reduced-order model \( \Sigma_j^z \) in (8) can be larger than that of the original full-order system \( \Sigma_j^z \) in (1).

**Proof:** Let us first consider the potential energies of the original and reduced systems

\[
\varphi_{x_j} = \frac{1}{2}z_j^T K_j z_j = \frac{1}{2}z_j^T P T_{j}^T K T_j P T_{j}^T z_j, \quad \varphi_x = \frac{1}{2}x^T K x
\]

which can be obtained by (8). We can then obtain the difference between the potential energy of the original and reduced system using (6) as following

\[
\varphi_{x}-\varphi_x = \frac{1}{2}x^T (\bar{T}_j - T_{j2}) (-2(T_{j2}^T K T_{j2} + Q)^{-1}T_{j2}^T K T_{j1}) x = \sum
\]

where \( [\bar{T}_j, T_{j2}] := \bar{T}_j, T_{j2} \in \mathbb{R}^{n \times m}, \bar{T}_j \in \mathbb{R}^{n \times (n-m)} \).

To show that \( \varphi_x \) can be larger than \( \varphi_{x_j} \), we assign the following vector to \( x \) s.t.,

\[
x = \bar{T}_j \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \bar{T}_j \begin{bmatrix} \sum_r 2(T_{j2}^T K T_{j2} + Q)^{-1}T_{j2}^T K T_{j1} \end{bmatrix}
\]

where \( r_1 \in \mathbb{R}^m \) is the non-zero arbitrary vector and \( Q \in \mathbb{R}^{(n-m) \times (n-m)} \) is the arbitrary positive definite matrix. Note that \( T_{j2}^T K T_{j2} + Q \) is always positive definite, so it is invertible because \( T_{j2}^T K T_{j2} \) is positive semi-definite and \( Q \) is positive definite. Then the energy difference is given by

\[
\varphi_x - \varphi_{x_j} = r_1^T \bar{T}_j T_{j1} K T_{j2} r_2 + \frac{1}{2}r_2^T T_{j2}^T K T_{j2} r_2 = -\frac{1}{2}r_2^T Q r_2
\]

This means that the potential energy of the reduced system can be larger than that of the original system (i.e., \( \varphi_x < \varphi_{x_j}, \forall r_2 \neq 0 \)). In the same way, we can show that the kinetic energy of the reduced system can be larger than that of the original system.

This Prop. 1 shows that the process of BMR (i.e., \( \Sigma_j^z \rightarrow \Sigma_j^z \)) can violate the passivity, and, consequently, that of the whole model switching in Fig. 1. At the same time, its proof exhibits the cause of this possible passivity violation is due to the off-diagonal terms in the matrix \( F \) in (9), and suggests to eliminate those off-diagonal terms for passive model switching in Fig. 1. For this, in this paper, we fuse the BMR with the simultaneous diagonalization of \( M \) and \( K \) of the full FEM dynamics (1) as below.
B. Passive Model Switching with Simultaneous Diagonalization

The simultaneous diagonalization problem [29] of $M, K$ in (1) is given by:

$$ (K - \gamma_q M)v_q = 0, \quad q = 1, 2, \ldots, n $$

(10)

where $\gamma_q \in \mathbb{R}, v_q \in \mathbb{R}^n$ are the generalized eigenvalues and eigenvectors. This problem (10) can be written as

$$ M^{-\frac{1}{2}}KM^{-\frac{1}{2}}M^\frac{1}{2}v_q = \gamma_q M^\frac{1}{2}v_q $$

which can be interpreted as the eigenvalue problem of the symmetric matrix $M^{-\frac{1}{2}}KM^{-\frac{1}{2}}$ with its eigenvectors $w_q := M^\frac{1}{2}v_q$ and can be rewritten as

$$ M^{-\frac{1}{2}}KM^{-\frac{1}{2}}W = WT, \quad WTW = I $$

(11)

where $W := [M^\frac{1}{2}v_1, \ldots, M^\frac{1}{2}v_n] = M^\frac{1}{2}V \in \mathbb{R}^{n \times n}$. $V := [v_1, v_2, \ldots, v_n] \in \mathbb{R}^{n \times n}$ and $\Gamma = \text{diag}(\gamma_1, \gamma_2, \ldots, \gamma_n) \in \mathbb{R}^{n \times n}$. If we use this $V \in \mathbb{R}^{n \times n}$ as the transformation matrix, we can simultaneously diagonalize $M$ and $K$ along the direction of each basis vector $v_q \in \mathbb{R}^n$ s.t.,

$$ V^TMV = W^TM^{-\frac{1}{2}}KM^{-\frac{1}{2}}W = WTW = I $$

$$ V^TKV = W^TM^{-\frac{1}{2}}KM^{-\frac{1}{2}}W = \Gamma $$

Now, we project the BMR of Sec. III-A into the basis vectors of this $V \in \mathbb{R}^{n \times n}$ to simultaneously diagonalize $M, K$, thereby, eliminating the (passivity-breaking) off-diagonal terms in the matrix $F$ in (9) both for the kinetic energy and the potential energy. This process can be written by:

$$ x_i := \tilde{T}_i P^Tz_i := : V\bar{\xi}_i $$

(12)

where $x_i \in \mathbb{R}^n$ is the "reconstructed" full state $x \in \mathbb{R}^n$ from the balancedly-reduced state $z_i \in \mathbb{R}^m$, and $\bar{\xi}_i \in \mathbb{R}^n$ is its projection into the simultaneously-diagonalizing basis vectors of $V \in \mathbb{R}^{n \times n}$. Here, we can always compute $\bar{\xi}_i$, since the matrix $V$ is non-singular. However, all the elements of $\bar{\xi}_i \in \mathbb{R}^n$ may not be necessary, since the source of information of (12) is only the $m$-dimensional $z_i$. To decide which components of $\bar{\xi}_i \in \mathbb{R}^n$ are necessary to capture this information, we first notice that the information content of each element of $z_i$ is captured by the first $m$ Hankel singular values of $\Lambda_i$ in (5). Then, utilizing the similarity between this Hankel norm and the covariance of PCA/POD [30], we can then compute the "covariance" of $\xi_i$ similar to the covariance propagation via a linear map of Kalman filtering [31]:

$$ C_{\xi_i} := V^{-\frac{1}{2}}\tilde{T}_i P^T\Lambda_i P\tilde{T}_i^T V^{-\frac{1}{2}} \in \mathbb{R}^{n \times n} $$

We then choose the reduced state $\xi_i \in \mathbb{R}^{m_{\xi_i}}$, as the collection of the elements of $\bar{\xi}_i$ with $m_{\xi_i}$-th largest diagonal values of $C_{\xi_i}$, i.e.,

$$ \xi_i := P_{\xi_i}\bar{\xi}_i = P_{\xi_i}V^{-\frac{1}{2}}x_i \in \mathbb{R}^{m_{\xi_i}} $$

(13)

where $P_{\xi_i} \in \mathbb{R}^{m_{\xi_i} \times n}$ is the selection matrix. The dimension $m_{\xi_i}$ is chosen to retain the information produced by $z_i \in \mathbb{R}^n$ as measured by $(\sum_{k=1}^{m_{\xi_i}} C_{\xi_i,kk})/(\sum_{k=1}^{n} C_{\xi_i,kk})$. In general, $m_{\xi_i} \geq m$, since the simultaneous diagonalization imposes additional constraints, although, in practice, we found $m_{\xi_i}$ still small for substantial simulation speed-up (see Sec. IV).

To decide which projection $\bar{\xi}_i$ to $\xi_i$ via (13), which even further enforces passivity of the model switching in Fig. 1. This is because the reconstruction process preserves the energy (i.e., $V_{\xi_i}(\Sigma_{\xi_i}^\xi) = V_{\xi}(\Sigma_{\xi}^\xi)$) as previously mentioned, and the reduction process discards some states after simultaneous diagonalization to eliminate the off-diagonal terms in the matrix $F$ in (9), so the energy always decreases (i.e., $V_{\xi}(\Sigma_{\xi}^\xi) \leq V_{\xi}(\Sigma_{\xi}^\xi)$). Notice also that $M_{\xi_i}$ is diagonal and $K_{\xi_i}$ has rather a simple structure, which further allows us to speed up the simulation computation.

<table>
<thead>
<tr>
<th>Method</th>
<th>Full-order model</th>
<th>Reduced-order model 1</th>
<th>Reduced-order model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard BMR</td>
<td>180</td>
<td>52</td>
<td>67</td>
</tr>
<tr>
<td>Proposed method</td>
<td>180</td>
<td>67</td>
<td>78</td>
</tr>
</tbody>
</table>

TABLE I: The number of states used in the simulation.
IV. SIMULATION AND EXPERIMENT RESULTS

A. Model Switching Simulation

To verify the efficacy of the proposed passive model reduction and switching framework, we conduct a simulation that switches the reduced-order model corresponding to the areas where the external force is applied. In this simulation, we choose an soft object as a thin plate made of silicone rubber with the system parameters as follow: 1) material properties $E = 145 MPa$, $\nu = 0.49$, $\rho = 2300 kg/m^3$ where $E$, $\nu$, $\rho$ are Young’s modulus, Poisson’s ratio, and the density; 2) length $(l_x, l_y, l_z) = (0.5, 0.2, 0.01)m$. Here, we fix one side of the plate to the environment to generate deformation.

The simulation is performed by applying standard BMR and proposed framework to switch the model. The number of states of the full-order and reduced-order models for each method is given in the Table I. The simulation results are presented in Fig. 2. Since the model switching between the reduced-order models obtained by standard BMR can violate the discrete-time passivity, the total energy of the system becomes larger than the input energy, so the simulation behavior diverges. On the other hand, the proposed passive model reduction and switching framework always guarantee the discrete-time passivity, so stable simulation is possible while switching the model corresponding to the areas where the input is applied.

B. Experiment Setup

We experiment to verify the performance of the proposed framework in Sec. III. The experiment environment consists of a 7-DOF robot manipulator (FRANKA EMIKA Panda), ATI Gamma force/torque (F/T) sensor, and soft object. See Fig. 3. The F/T sensor is attached to the robot end-effector to measure the external force acting on the soft object at the contact location. Here, we connect the F/T sensor to a part of the soft object using thin thread to apply the force on only a small number of nodes in a thin area, and we assume that the external force acting on each node acts uniformly on the nodes.

For a soft object, we use soft material made of silicone rubber and fix both sides of the soft object to the environment to generate large deformation. Then the torque-controlled robot manipulator moves the thread connected to the soft object while the thread is held tight. For this, we design the end-effector impedance controller [32].

Also, we attach the markers to a part of the soft object to measure the ground-truth deformation to compare with simulation result using OptiTrack motion capture system.

C. Simulation Result with Experimental data

To verify the performance of our proposed fast soft object simulation with passive model reduction and switching framework, we obtain the experimental data by performing an experiment in which deformation occurred by applying a force to two different contact locations. Then, we compare the deformation of simulation with the experimentally measured ground-truth deformation at the point where the markers are attached.

For the simulation of a soft object, we model the soft object as a cylinder with length 0.6m and diameter 1.5cm using the FEM model in Sec. II-A with a mesh of elements (tetrahedrons) connecting a set of 3D nodes. Here, we use a total of 3842 nodes and 13985 tetrahedrons. Then, we obtain the reduced-order models for each contact location to achieve fast computation speed while capturing the system dynamics with a low number of states. The number of states of full-order and reduced-order model according to each contact location is given in Table II. The material properties related to the dynamics of the object are determined by the optimization process that finds the parameters which best match the simulated deformation with the experimental deformation data obtained by applying various contact forces at the known locations where the marker is attached. The obtained material properties are $E = 3.45 MPa$, $\nu = 0.475$, $\rho = 1187.4 kg/m^3$ which are within range of the known properties of the silicone rubber.

The performance of the simulation is shown in Fig. 4, 5, and Table III. As shown in Fig. 4 which shows experimental and simulated deformation at the locations with markers and Fig. 5 which shows the deformed shape of the object in experiment and simulation, not only the full-order model but also the reduced-order model represents the behavior of the experiment with some reasonable accuracy. In particular, the reduced-order model has an accuracy similar to that of the full-order model, even though only a small number of states

<table>
<thead>
<tr>
<th>Model</th>
<th>Full-order model</th>
<th>Reduced-order model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of states</td>
<td>11,526</td>
<td>164 (1st contact)</td>
</tr>
<tr>
<td>Computation speed</td>
<td>5.2 Hz</td>
<td>420 Hz</td>
</tr>
</tbody>
</table>

TABLE II: The number of states and computation speed in simulation using full-order and reduced-order model.

<table>
<thead>
<tr>
<th>Root Mean Square Error</th>
<th>Marker 1</th>
<th>Marker 2</th>
<th>Marker 3</th>
<th>Marker 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment vs Full-order model</td>
<td>0.34 cm</td>
<td>1.08 cm</td>
<td>1.34 cm</td>
<td>0.56 cm</td>
</tr>
<tr>
<td>Experiment vs Reduced-order model</td>
<td>0.38 cm</td>
<td>1.11 cm</td>
<td>1.38 cm</td>
<td>0.58 cm</td>
</tr>
</tbody>
</table>

TABLE III: Simulation accuracy versus experiment at the location of each marker for full-order and reduced-order model.
V. Conclusions

We propose a novel fast simulation framework for soft objects (and possibly robots) with intermittent contacts. For this, for each intermittent contact mode, we perform a balanced model reduction of the full-order FEM model with the contact force as the input and the shape of the soft object as the output. We then switch among these reduced-order models (each with its contact mode) to address the changing locations of the intermittent contacts. To ensure the stability of the whole simulation, we render this switching among different reduced-order models (with different contact mode) to be passive. The devised theory is then experimentally demonstrated. Some possible future research topics include: 1) extension to non-intermittent and general contacts; 2) extension to FEM with nonlinear kinematics and material properties; 3) application for soft object robotic manipulation and soft robot control.

References


