Passive Decomposition Approach to Formation and Maneuver Control of Multiple Rigid Bodies

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A passive decomposition framework for the formation and maneuver controls for multiple rigid bodies is proposed. In this approach, the group dynamics of the multiple agents is decomposed into two decoupled systems: The shape system representing internal group formation shape (formation, in short), and the locked system abstracting the overall group maneuver as a whole (maneuver, in short). The decomposition is natural in that the shape and locked systems have dynamics similar to the mechanical systems, and the total energy is preserved. The shape and locked system can be decoupled without the use of net energy. The decoupled shape and locked systems can be controlled individually to achieve the desired formation and maneuver tasks. Since all agents are given equal status, the proposed scheme enforces a group coherence among the agents. By abstracting a group maneuver by its locked system whose dynamics is similar to that of a single agent, a hierarchical control structure for the multiple agents can be easily imposed in the proposed framework. A decentralized version of the controller is also proposed, which requires only undirected line communication (or sensing) graph topology.
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1 Introduction

In many applications, a team of multiple agents holds eminent promises to achieve a level of performance, capability, and robustness beyond what a single agent can provide. Some examples include space interferometry [1–3], reconnaissance/surveillance using multiple UAVs (uninhibited aerial vehicles) [4], underwater assessment [5], mobile sensor network [6], and multirobot cooperation [7,8], to name a few.

A basic requirement for such multi-agent systems is to achieve certain desired internal group formation and overall group maneuver at the same time. As a simple example, consider two particles moving on the x-axis. A desired group behavior can be achieved by controlling their formation (e.g., x_1-x_2) and their maneuver (e.g., $\frac{1}{2}(x_1+x_2)$), simultaneously, where $x_i \in \mathrm{Re}$ is the x-position of the ith particle ($i=1,2$). Many control schemes have been proposed for this multi-agent formation control problem, and, according to [1], most of them can be classified into three categories: Behavior-based approach, leader-follower approach, and virtual-structure approach.

In the behavior-based approach [7–11], each agent's control is designed to be the sum of (or switchings among) several prescribed behaviors invoked by local external stimuli (e.g., distances to neighboring agents). The dynamics are designed such that (s.t.), collectively, certain desired formation and maneuver behavior will emerge. Unfortunately, the convergence/stability proof of such collective behavior is often very complicated and difficult. Moreover, typically, only equilibria that correspond to simple behaviors (e.g., constant heading [9]) are provable. These approaches are, therefore, not easily adaptable for complicated formations and maneuvers (e.g., time-varying formation).

In the leader-follower approach [12–15], an agent of a group is designated as the group leader and its motion alone represents the overall group maneuver. The leader's motion is controlled to achieve a desired maneuver, while a desired formation is achieved by controlling the remaining agents (followers) to maintain certain relative distances with respect to (w.r.t.) the leader. The main drawback of the usual leader-follower approach is that a leader has no feedback from the followers (i.e., formation feedback [1] is absent). Thus, the group can show such incoherent behaviors as run-away of a fast leader with slower followers left behind. Another consequence of this unidirectional feedback is poor disturbance rejection (e.g., a disturbance on the leader can be amplified while propagating throughout the followers [16]).

In the virtual-structure approach [5,17–19], a virtual dynamical system (simulated in software) is used to abstract the group maneuver. To avoid the incoherent group behavior of the leader-follower approach, they incorporate formation feedback into the virtual system's dynamics s.t. its evolution is also affected by each agent's behavior. The desired maneuver and formation are then achieved by controlling the virtual system and agents as a leader and followers. However, simulation of such a virtual system (that often requires additional differential equation solvers) might restrict applicability of this approach when available computing power is scarce (e.g., distributed control implementation). More fundamentally, by relying on artificially simulated dynamics rather than real agents' states, this virtual system may not capture real group maneuver. For example, consider a virtual point mass which, through a kinematic feedback (i.e., formation feedback), is coupled with a group of point masses that are revolving on a circle periodically. Then, in general, the orbit and phase of this virtual mass would be different than those of the overall group (e.g., group center of mass). As this difference becomes larger (e.g., with smaller radius and higher speed), it would become less suitable to use this virtual system to abstract the overall group maneuver.

In this paper, a passive decomposition approach is proposed for the formation and maneuver control of multiple rigid bodies (e.g., satellites, submarines, and robots). Passive decomposition was originally proposed for tele-operators [20,21] and was later generalized for general mechanical systems [22,23]. In this paper,

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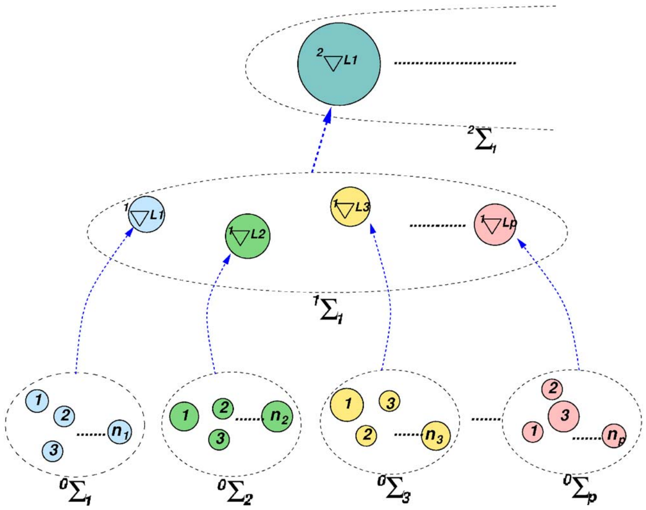

Fig. 1 Hierarchy by abstraction: Each group $^0\Sigma_i$ abstracted by its locked system $^1\nabla^{Li}$; group of groups $^1\Sigma_1$ again abstracted by its locked system $^2\nabla^{L1}$; formation among agents/groups described by their shape systems (not shown)

using the passive decomposition, the actual multi-agent six-degree-of-freedom nonlinear rigid-body dynamics are decomposed into: (1) The *shape system* describing the internal group formation, and (2) the *locked system* abstracting the overall group maneuver. By controlling the decoupled locked and shape systems individually, the desired maneuver and formation can then, respectively, be achieved. The shape and locked systems have dynamics similar to that of mechanical systems. Additionally, the total energy of the group's agents is preserved in the energies of the shape and locked systems. Furthermore, the shape and locked systems can be decoupled from each other in an energetically passive manner (i.e., does not require input of net energy). Recently, the authors in [3] captured a similar idea of formation and maneuver decomposition. However, their decomposition is not passive, and is only applicable for linear dynamics (e.g., translational dynamics). In contrast, our passive decomposition is equally applicable to linear and nonlinear dynamics (e.g., attitude dynamics) and have the advantageous passivity property.

Since the passive decomposition allows the locked and shape systems to be perfectly decoupled from each other, the proposed control can achieve tight formation keeping and precise maneuver management without any dynamic crosstalk between them. With this, we can avoid the situations where the internal formation wriggles as the overall group accelerates, or the overall group motion fluctuates as the formation expands/contracts. Formation and maneuver decoupling is not achieved in previous approaches with the exception of [12–15], which, in essence, rely on often-expensive acceleration feedback. In contrast, our decoupling requires only position and velocity information, which are more readily available.

The locked system abstracts the group maneuver similar to the virtual system in the virtual-structure approach. However, since the locked system (and also the shape system) is directly derived from the agents' real states and dynamics, our abstraction is free from the aforementioned problems of the virtual-structure approach due to the artificiality. In addition, our framework provides formation feedback so that the group together will automatically slow down or speed up to keep pace with slower or faster agents. Because the locked system itself has dynamics similar to that of a single agent, a group of grouped agents can, in turn, be aggregated into formation by considering the passive decomposition of the set of locked systems. In this way, a hierarchical grouping of agents can be obtained in a very natural way (Fig. 1). This hierarchy would be useful for controlling a collection of a large number of agents and groups.

One remarkable property of the passive decomposition is that the locked and shape decoupling does not change the total energy of the system. This implies that the (energetic) passivity [24] of the group's open-loop dynamics is not altered by the decoupling. A consequence of this is that, as one agent is actuated or mechanically perturbed by its environment, the decoupling control serves to distribute this energy input from the environment among other agents. Such passivity property would be useful for orbital formation flying, as formation and maneuver can then be decoupled without affecting the orbits and the periodicity of the group motion. This passivity property would also enhance the coupling stability [25] of the group when it interacts with other groups to form a larger group (e.g., through the hierarchy in Fig. 1) or physically interacts with real external systems (e.g., human astronaut or robonaut [26]). In this paper, we focus mainly on the formation and maneuver aspects of the passive decomposition. For applications where this passivity property is emphasized, refer to [20,21,27].

Direct implementation of the proposed control scheme generally requires the state information of all the agents (e.g., via centralized communication). To relax this communication requirement, a decentralized controller is also proposed here, which only requires undirected line communication (or sensing) graph topology among the agents. We also show that this decentralized control has performance similar to that of the centralized control when desired group behavior is not too aggressive.

The rest of the paper is organized as follows. The control problem is formulated in Sec. 2. The passive decomposition is derived in Sec. 3. Design of the centralized control is given in Sec. 4, and its decentralization is presented in Sec. 5. Section 6 presents an extension of the results to the equivalence class of the formation variables and Sec. 7 presents the simulation results. Section 8 contains some concluding remarks.

2 Problem Formulation

2.1 System Modeling. We consider a group of m-agents that have the dynamics of a fully actuated 6-DOF flying rigid body [19]. The translational dynamics of the ith agent with respect to a common inertial frame $\mathcal{F}_o$ is then given by the following 3-DOF linear point-mass dynamics: $i=1,\ldots,m$,

$$m_i\ddot{\mathbf{x}}_i = \mathbf{t}_i + \mathbf{f}_i \tag{1}$$

where $m_i \in \mathrm{Re}^+$ is the (constant) mass, and $\mathbf{x}_i=[x_i,y_i,z_i]^T \in \mathrm{Re}^3$, $\mathbf{t}_i \in \mathrm{Re}^3$, and $\mathbf{f}_i \in \mathrm{Re}^3$ are the position, control (to be designed), and environmental disturbance (e.g., gravitational force, drag, etc.) w.r.t. $\mathcal{F}_o$, respectively.

Following [28,29], we also model the attitude dynamics of the ith agent w.r.t. the inertia frame $\mathcal{F}_o$ by the following 3-DOF nonlinear dynamics [30]: $i=1,\ldots,m$,

$$\mathbf{H}_i(\boldsymbol{\zeta}_i)\dot{\boldsymbol{\omega}}_i + \mathbf{Q}_i(\boldsymbol{\zeta}_i,\boldsymbol{\omega}_i)\boldsymbol{\omega}_i = \boldsymbol{\tau}_i + \boldsymbol{\delta}_i \tag{2}$$

where $\boldsymbol{\zeta}_i=[\phi_i,\theta_i,\psi_i]^T \in \mathrm{Re}^3$ are the roll, pitch, and yaw angles with $-\pi/2<\theta_i<\pi/2$, $\boldsymbol{\omega}_i(t)=(d/dt)\boldsymbol{\zeta}_i(t) \in \mathrm{Re}^3$ is the angular rate, $\boldsymbol{\tau}_i,\boldsymbol{\delta}_i \in \mathrm{Re}^3$ are the control (to be designed) and disturbance w.r.t. $\mathcal{F}_o$, respectively. Also, $\mathbf{H}_i(\boldsymbol{\zeta}_i) \in \mathrm{Re}^{3\times 3}$ and $\mathbf{Q}_i(\boldsymbol{\zeta}_i,\dot{\boldsymbol{\zeta}}_i) \in \mathrm{Re}^{3\times 3}$ are the symmetric and positive-definite inertia matrix and the Coriolis matrix s.t. $\dot{\mathbf{H}}_i(\boldsymbol{\zeta}_i)-2\mathbf{Q}_i(\boldsymbol{\zeta}_i,\dot{\boldsymbol{\zeta}}_i)$ is skew symmetric. In this paper, we mainly consider the agents whose translation and attitude dynamics are derived w.r.t. a common inertial frame $\mathcal{F}_o$ as in (1) and (2). How to extend the presented results in coordinates to the case where each agent's dynamics is given w.r.t. its own body frame (or other frames) is a topic for future research.

2.2 Formation and Maneuver Control Objectives. For the group of m-agents (1) and (2), we define the formation and maneuver variables s.t.

$$\mathbf{p}_E(\mathbf{x}_1(t),\mathbf{x}_2(t),\ldots,\mathbf{x}_m(t),\boldsymbol{\zeta}_1(t),\boldsymbol{\zeta}_2(t),\ldots,\boldsymbol{\zeta}_m(t)) \in \mathrm{Re}^{6m-p} \quad (3)$$

$$\mathbf{p}_L(\mathbf{x}_1(t),\mathbf{x}_2(t),\ldots,\mathbf{x}_m(t),\boldsymbol{\zeta}_1(t),\boldsymbol{\zeta}_2(t),\ldots,\boldsymbol{\zeta}_m(t)) \in \mathrm{Re}^{p} \quad (4)$$

where $\mathbf{p}_E:\mathrm{Re}^{6m}\to\mathrm{Re}^{6m-p}$ and $\mathbf{p}_L:\mathrm{Re}^{6m}\to\mathrm{Re}^{p}$ are smooth maps with full rank Jacobian (i.e., smooth submersion) and p is a positive integer less than $6m$. We suppose that $\mathbf{p}_E$ and $\mathbf{p}_L$ are designed s.t. for a given mission purpose, they can represent formation and maneuver aspects among the group agents, respectively. The formation and maneuver control objectives can then be written as

$$\mathbf{p}_L(t) \to \mathbf{p}_L^d(t) \quad (5)$$

$$\mathbf{p}_E(t) \to \mathbf{p}_E^d(t) \quad (6)$$

where $\mathbf{p}_E^d(t)\in\mathrm{Re}^{6m-p}$, $\mathbf{p}_L^d(t)\in\mathrm{Re}^{p}$ are desired formation and maneuver trajectories, respectively.

For an illustration, let us consider two agents having 3-DOF motion (translation and yaw rotation) on the (x,y)-plane. One example of the formation and maneuver variables pair is then

$$\mathbf{p}_E(t) := \begin{pmatrix} x_1 - x_2 - L\cos(\psi_1) \\ y_1 - y_2 - L\sin(\psi_1) \\ \psi_1 - \psi_2 \end{pmatrix} \in \mathrm{Re}^3 \quad \text{and}$$

$$\mathbf{p}_L(t) := \begin{pmatrix} \frac{1}{2}(x_1+x_2) \\ \frac{1}{2}(y_1+y_2) \\ \frac{1}{2}(\psi_1+\psi_2) \end{pmatrix} \in \mathrm{Re}^3 \quad (7)$$

where x_i, y_i, ψ_i are the positions and the yaw angle of the ith agent $(i=1,2)$, and L is a positive scalar defining distance between them. If we achieve the formation objective (6) with $\mathbf{p}_E^d(t)=\mathbf{0}$, then the two agents will have aligned yaw angles and form a rod-like shape of length L, whose orientation is determined by the first agent's yaw angle. In addition, by enforcing the maneuver objective (5), we can drive this rod's 3-DOF configuration (i.e., location on the (x,y)-plane and orientation) as specified by the target trajectory $\mathbf{p}_L^d(t)$.

In this paper, we consider separate formation and maneuver variables for the translation and attitude dynamics (1) and (2) s.t. $\mathbf{p}_L(t)=[\mathbf{x}_L^T(t),\boldsymbol{\zeta}_L^T(t)]^T\in\mathrm{Re}^6$ and $\mathbf{p}_E(t)=[\mathbf{x}_E^T(t),\boldsymbol{\zeta}_E^T(t)]^T\in\mathrm{Re}^{6(m-1)}$, where $\mathbf{x}_\star$ and $\boldsymbol{\zeta}_\star$ $(\star\in\{L,E\})$ are functions of $(\mathbf{x}_1,\ldots,\mathbf{x}_m)$ and $(\boldsymbol{\zeta}_1,\ldots,\boldsymbol{\zeta}_m)$, respectively. In particular, we use the following simple but versatile formation variables:

$$\mathbf{x}_E := [\mathbf{x}_1^T-\mathbf{x}_2^T,\mathbf{x}_2^T-\mathbf{x}_3^T,\ldots,\mathbf{x}_{m-1}^T-\mathbf{x}_m^T]^T \in \mathrm{Re}^{3(m-1)} \quad (8)$$

$$\boldsymbol{\zeta}_E := [\boldsymbol{\zeta}_1^T-\boldsymbol{\zeta}_2^T,\boldsymbol{\zeta}_2^T-\boldsymbol{\zeta}_3^T,\ldots,\boldsymbol{\zeta}_{m-1}^T-\boldsymbol{\zeta}_m^T]^T \in \mathrm{Re}^{3(m-1)} \quad (9)$$

i.e., relative positions and attitudes between two consecutive agents. The maneuver variables $\mathbf{x}_L(t)\in\mathrm{Re}^3$ and $\boldsymbol{\zeta}_L(t)\in\mathrm{Re}^3$ will be defined in Sec. 4 so that their dynamics are decoupled from the dynamics of the formation variables (8) and (9). $\mathbf{p}_L^d(t),\mathbf{p}_E^d(t)$ in (5) and (6) can also then be partitioned s.t. $\mathbf{p}_L^d(t)=[\mathbf{x}_L^{dT}(t),\boldsymbol{\zeta}_L^{dT}(t)]^T\in\mathrm{Re}^6$ and $\mathbf{p}_E^d(t)=[\mathbf{x}_E^{dT}(t),\boldsymbol{\zeta}_E^{dT}(t)]^T\in\mathrm{Re}^{6(m-1)}$ where $\mathbf{x}_L^d(t),\boldsymbol{\zeta}_L^d(t)\in\mathrm{Re}^3$ and $\mathbf{x}_E^d(t),\boldsymbol{\zeta}_E^d(t)\in\mathrm{Re}^{3(m-1)}$ are desired maneuver and formation for the group translation and attitude, respectively. Of course, other structures for (8) and (9) would also be possible. See Sec. 6 for an extension of the results to a certain equivalence class of the formation variables (8) and (9).

The structures of (8) and (9) (and those in Sec. 6) are chosen for the following reasons: (1) With these formation variables, as to be shown in Sec. 3, we can achieve a simple (closed-form) expression for the passive decomposition with which concepts can be presented more efficiently and clearly; and (2) we can exploit the flat properties of the translation dynamics (1) (i.e., Riemannian curvature vanishes everywhere [31]) and of the formation variable (8) (i.e., its level sets are given by flat planes). With these flat properties, as shown in Sec. 4.1, the locked system of the translation dynamics (1) will have the usual point mass dynamics whose configuration and mass are given by the position of the center of mass and the sum of all agents' mass, respectively. As shown in Sec. 5, the structures of the formation variables (8) and (9) implicitly define the required communication topology.

Remark 1. Using the coordinate-independent results of [22,23], instead of (1), (2), (8), and (9), we can also incorporate different formulations of the group dynamics (e.g., derived w.r.t. an orbiting frame or unit quaternion for $SO(3)$) and general holonomic constraints (e.g., $\mathbf{p}_L,\mathbf{p}_E$ in (3) and (4)), respectively. However, expressions for the passive decomposition and control might be complicated.

3 The Passive Decomposition

In this section, we use the passive decomposition [22,23] to decompose the multi-agent group dynamics into two decoupled systems: The shape system representing internal group formation as given by the formation variables [8] and [9], and the locked system abstracting overall group maneuver.

To derive a unified decomposition for the translation and attitude dynamics (1) and (2) under the formation variables (8) and (9), we consider the following group dynamics of m n-DOF mechanical systems:

$$\begin{aligned}
\mathbf{M}_1(\mathbf{q}_1)\ddot{\mathbf{q}}_1 + \mathbf{C}_1(\mathbf{q}_1,\dot{\mathbf{q}}_1)\dot{\mathbf{q}}_1 &= \mathbf{T}_1 + \mathbf{F}_1 \\
\mathbf{M}_2(\mathbf{q}_2)\ddot{\mathbf{q}}_2 + \mathbf{C}_2(\mathbf{q}_2,\dot{\mathbf{q}}_2)\dot{\mathbf{q}}_2 &= \mathbf{T}_2 + \mathbf{F}_2 \\
&\vdots \\
\mathbf{M}_m(\mathbf{q}_m)\ddot{\mathbf{q}}_m + \mathbf{C}_m(\mathbf{q}_m,\dot{\mathbf{q}}_m)\dot{\mathbf{q}}_m &= \mathbf{T}_m + \mathbf{F}_m
\end{aligned} \quad (10)$$

under the following formation variable (holonomic constraint)

$$\mathbf{q}_E^T := [\mathbf{q}_1^T-\mathbf{q}_2^T,\mathbf{q}_2^T-\mathbf{q}_3^T,\ldots,\mathbf{q}_{m-1}^T-\mathbf{q}_m^T]^T \in \mathrm{Re}^{(m-1)n} \quad (11)$$

where $\mathbf{q}_i\in\mathrm{Re}^n$ and $\dot{\mathbf{q}}_i\in\mathrm{Re}^n$ are the configuration and the velocity, $\mathbf{T}_i\in\mathrm{Re}^n$ and $\mathbf{F}_i\in\mathrm{Re}^n$ are the controls and the disturbances, and $\mathbf{M}_i(\mathbf{q}_i)\in\mathrm{Re}^{n\times n}$ and $\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\in\mathrm{Re}^{n\times n}$ are the symmetric and positive-definite inertia matrix and the Coriolis matrix s.t. $(d/dt)\mathbf{M}_i(\mathbf{q}_i)-2\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)$ are skew symmetric, respectively.

Following the tangent-space decomposition in [22,23,32], the velocity (i.e., tangent vector) $\dot{\mathbf{q}}:=[\dot{\mathbf{q}}_1^T,\dot{\mathbf{q}}_2^T,\ldots,\dot{\mathbf{q}}_m^T]^T\in\mathrm{Re}^{mn}$ of the group dynamics (10) can then be decomposed into the locked system velocity $\mathbf{v}_L\in\mathrm{Re}^n$ and the shape system velocity $\mathbf{v}_E\in\mathrm{Re}^{(m-1)n}$ s.t.

$$\begin{pmatrix} \mathbf{v}_L \\ \uparrow \\ | \\ \mathbf{v}_E \\ | \\ \downarrow \end{pmatrix} := \underbrace{\begin{bmatrix} \phi_1(\mathbf{q}) & \phi_2(\mathbf{q}) & \phi_3(\mathbf{q}) & \cdots & \phi_{m-1}(\mathbf{q}) & \phi_m(\mathbf{q}) \\ \mathbf{I} & -\mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & -\mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \ddots & -\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \cdots & \mathbf{I} & -\mathbf{I} \end{bmatrix}}_{=:\mathbf{S}(\mathbf{q}) \in \mathrm{Re}^{mn\times mn}} \begin{pmatrix} \dot{\mathbf{q}}_1 \\ \dot{\mathbf{q}}_2 \\ \dot{\mathbf{q}}_3 \\ \vdots \\ \vdots \\ \dot{\mathbf{q}}_m \end{pmatrix} \tag{12}$$

where $\mathbf{q} := [\mathbf{q}_1^T, \mathbf{q}_2^T, \ldots, \mathbf{q}_m^T]^T \in \mathrm{Re}^{mn}$ and $\phi_i(\mathbf{q}) \in \mathrm{Re}^{n\times n}$ $(i = 1, \ldots, m)$ is given by

$$\phi_i(\mathbf{q}) := [\mathbf{M}_1(\mathbf{q}_1) + \mathbf{M}_2(\mathbf{q}_2) + \ldots + \mathbf{M}_m(\mathbf{q}_m)]^{-1}\mathbf{M}_i(\mathbf{q}_i) \tag{13}$$

Using the fact that $\phi_i(\mathbf{q})$ in (13) is nonsingular and the property that

$$\phi_1(\mathbf{q}_1) + \phi_2(\mathbf{q}_2) + \cdots + \phi_m(\mathbf{q}_m) = \mathbf{I} \tag{14}$$

we can show that $\mathbf{S}(\mathbf{q})$ in (12) is nonsingular.

From (12), the shape system velocity $\mathbf{v}_E \in \mathrm{Re}^{(m-1)n}$ is given by

$$\mathbf{v}_E(t) = \frac{d}{dt}\mathbf{q}_E(\mathbf{q}_1(t), \mathbf{q}_2(t), \ldots, \mathbf{q}_m(t)) \tag{15}$$

i.e., time derivative of the formation variable $\mathbf{q}_E$ in (11). Thus, having $\mathbf{q}_E$ as its configuration, the shape system would explicitly describe the formation aspect. In addition, with the definition (13), the locked system velocity $\mathbf{v}_L$ in (12) is given by

$$\mathbf{v}_L = \left[\sum_{i=1}^{m} \mathbf{M}_i(\mathbf{q}_i)\right]^{-1} [\mathbf{M}_1(\mathbf{q}_1)\dot{\mathbf{q}}_1 + \mathbf{M}_2(\mathbf{q}_2)\dot{\mathbf{q}}_2 \cdots + \mathbf{M}_m(\mathbf{q}_m)\dot{\mathbf{q}}_m] \tag{16}$$

i.e., average of all agents' velocities with each agent's inertia being the weighting. Notice that if the inertia matrices $\mathbf{M}_i(\mathbf{q}_i)$ are all scalar constants, this $\mathbf{v}_L$ in (16) will become the velocity of center of mass, i.e.,

$$\mathbf{v}_L = \frac{d}{dt}\left(\frac{\mathbf{M}_1\mathbf{q}_1 + \mathbf{M}_2\mathbf{q}_2 + \ldots \mathbf{M}_m\mathbf{q}_m}{\mathbf{M}_1 + \mathbf{M}_2 + \ldots \mathbf{M}_m}\right)$$

Therefore, with $\mathbf{v}_L$ in (16) as its velocity, the locked system would abstract overall group maneuver.

We also define the compatible decompositions of (12) s.t.

$$\begin{pmatrix} \mathbf{T}_L \\ \uparrow \\ \mathbf{T}_E \\ \downarrow \end{pmatrix} := \mathbf{S}^{-T}(\mathbf{q}) \begin{pmatrix} \mathbf{T}_1 \\ \mathbf{T}_2 \\ \vdots \\ \mathbf{T}_m \end{pmatrix} \text{ and } \begin{pmatrix} \mathbf{F}_L \\ \uparrow \\ \mathbf{F}_E \\ \downarrow \end{pmatrix} := \mathbf{S}^{-T}(\mathbf{q}) \begin{pmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \\ \vdots \\ \mathbf{F}_m \end{pmatrix} \tag{17}$$

where $\mathbf{S}^{-T}(\mathbf{q}) = (\mathbf{S}^{-1}(\mathbf{q}))^T$, and $\mathbf{F}_L, \mathbf{T}_L \in \mathrm{Re}^n$ and $\mathbf{F}_E, \mathbf{T}_E \in \mathrm{Re}^{(m-1)n}$ are the effects of environmental forcing and controls on the locked and shape systems, respectively. Here in (17), the matrix $\mathbf{S}^{-1}(\mathbf{q}) \in \mathrm{Re}^{mn\times mn}$ is found to be

$$\mathbf{S}^{-1}(\mathbf{q}) = \begin{bmatrix} \mathbf{I} & \Sigma_2(\mathbf{q}) & \Sigma_3(\mathbf{q}) & \cdots & \Sigma_m(\mathbf{q}) \\ \mathbf{I} & \Sigma_2(\mathbf{q}) - \mathbf{I} & \Sigma_3(\mathbf{q}) & \cdots & \Sigma_m(\mathbf{q}) \\ \mathbf{I} & \Sigma_2(\mathbf{q}) - \mathbf{I} & \Sigma_3(\mathbf{q}) - \mathbf{I} & \ddots & \Sigma_m(\mathbf{q}) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{I} & \Sigma_2(\mathbf{q}) - \mathbf{I} & \Sigma_3(\mathbf{q}) - \mathbf{I} & \cdots & \Sigma_m(\mathbf{q}) - \mathbf{I} \end{bmatrix} \tag{18}$$

where

$$\Sigma_i(\mathbf{q}) = \phi_i(\mathbf{q}) + \phi_{i+1}(\mathbf{q}) + \cdots + \phi_m(\mathbf{q}) \in \mathrm{Re}^{n\times n} \tag{19}$$

so that $\Sigma_1(\mathbf{q}) = \mathbf{I}$ and $\Sigma_m(\mathbf{q}) = \phi_m(\mathbf{q})$ from the definition of $\phi_i(\mathbf{q})$ in (13).

With the decomposition in (12), the group inertia of (10) is now block diagonalized s.t.

$$\mathbf{S}^{-T}(\mathbf{q})\mathrm{diag}[\mathbf{M}_1(\mathbf{q}_1), \mathbf{M}_2(\mathbf{q}_2), \cdots \mathbf{M}_m(\mathbf{q}_m)]\mathbf{S}^{-1}(\mathbf{q}) =: \begin{bmatrix} \mathbf{M}_L(\mathbf{q}) & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_E(\mathbf{q}) \end{bmatrix} \tag{20}$$

where $\mathbf{M}_L(\mathbf{q}) \in \mathrm{Re}^{n\times n}$ and $\mathbf{M}_E(\mathbf{q}) \in \mathrm{Re}^{(m-1)n\times(m-1)n}$ are the (symmetric and positive definite) inertia matrices for the locked and shape systems, respectively. This inertia block-diagonalizing property (20) implies that the total kinetic energy of the m-mechanical systems (10) is also decomposed into the sum of those of the locked and the shape systems (see Proposition 1 below). From (20) with (18), we can show that

$$\mathbf{M}_L(\mathbf{q}) := \mathbf{M}_1(\mathbf{q}_1) + \mathbf{M}_2(\mathbf{q}_2) + \cdots. + \mathbf{M}_m(\mathbf{q}_m) \in \mathrm{Re}^{n\times n} \tag{21}$$

i.e., the locked system inertia is given by the sum of those of all agents.

Using the decomposition (12) and (17) with the inertia block-diagonalizing property (20), the group dynamics (10) can then be partially decoupled s.t.

$$\underbrace{\mathbf{M}_L(\mathbf{q})\dot{\mathbf{v}}_L + \mathbf{C}_L(\mathbf{q},\dot{\mathbf{q}})\mathbf{v}_L}_{\text{locked system dynamics}} + \underbrace{\mathbf{C}_{LE}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_E}_{\text{coupling}} = \mathbf{T}_L + \mathbf{F}_L \tag{22}$$

$$\underbrace{\mathbf{M}_E(\mathbf{q})\ddot{\mathbf{q}}_E + \mathbf{C}_E(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_E}_{\text{shape system dynamics}} + \underbrace{\mathbf{C}_{EL}(\mathbf{q},\dot{\mathbf{q}})\mathbf{v}_L}_{\text{coupling}} = \mathbf{T}_E + \mathbf{F}_E \tag{23}$$

where we use the following definition

$$\begin{bmatrix} \mathbf{C}_L & \mathbf{C}_{LE} \\ \mathbf{C}_{EL} & \mathbf{C}_E \end{bmatrix} := \mathbf{S}^{-T}\mathrm{diag}[\mathbf{M}_1, \mathbf{M}_2, \cdots \mathbf{M}_m]\frac{d}{dt}(\mathbf{S}^{-1}) + \mathbf{S}^{-T}\mathrm{diag}[\mathbf{C}_1, \mathbf{C}_2, \cdots \mathbf{C}_m]\mathbf{S}^{-1} \tag{24}$$

with arguments being omitted for brevity.

We call the n-DOF system in (22) the *locked system*, which abstracts the overall group maneuver with $\mathbf{v}_L$ in (16) and $\mathbf{M}_L(\mathbf{q})$ in (21) as its velocity and inertia, respectively. Using (17) and (18), we can show that $\mathbf{F}_L = \mathbf{F}_1 + \mathbf{F}_2 + \cdots + \mathbf{F}_m$, i.e., the effects of external disturbances on the locked system is given by the sum of those of the individual agents. We also call the $(m-1)n$-DOF system in (23) the *shape system*, which explicitly describes internal group formation by having the formation variable $\mathbf{q}_E$ (11) as its configuration. Due to the inertia block-diagonalizing property (20), the couplings between the locked and shape systems are only through the terms $\mathbf{C}_{LE}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_E$ and $\mathbf{C}_{EL}(\mathbf{q},\dot{\mathbf{q}})\mathbf{v}_L$ in (22) and (23), that are functions of $\mathbf{q}$ and $\dot{\mathbf{q}}$. Thus, the locked and shape systems can be decoupled without utilizing acceleration feedback.

PROPOSITION 1. *The partially decoupled dynamics* (22) and (23) *have the following properties:*

1. $\mathbf{M}_L(\mathbf{q})$ *and* $\mathbf{M}_E(\mathbf{q})$ *are symmetric and positive definite. Moreover, total kinetic energy of the group (10) is decomposed into the sum of those of the shape and locked systems, s.t.*

$$\kappa(t) := \sum_{i=1}^{m} \frac{1}{2}\dot{\mathbf{q}}_i^T \mathbf{M}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i^T = \frac{1}{2}\mathbf{v}_L^T \mathbf{M}_L(\mathbf{q})\mathbf{v}_L + \frac{1}{2}\dot{\mathbf{q}}_E^T \mathbf{M}_E(\mathbf{q})\dot{\mathbf{q}}_E \tag{25}$$

2. $\dot{\mathbf{M}}_L(\mathbf{q}) - 2\mathbf{C}_L(\mathbf{q},\dot{\mathbf{q}})$ *and* $\dot{\mathbf{M}}_E(\mathbf{q}) - 2\mathbf{C}_E(\mathbf{q},\dot{\mathbf{q}})$ *are skew symmetric;*
3. $\mathbf{C}_{LE}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{C}_{EL}^T(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{0}$*;*
4. *Total environmental and control supply rates are decomposed into the sum of those of the locked and shape systems, i.e.,*

$$s_p(t) := \sum_{i=1}^{m} \mathbf{F}_i^T \dot{\mathbf{q}}_i = \mathbf{F}_L^T \mathbf{v}_L + \mathbf{F}_E^T \dot{\mathbf{q}}_E \quad \text{and}$$

$$s_c(t) := \sum_{i=1}^{m} \mathbf{T}_i^T \dot{\mathbf{q}}_i = \mathbf{T}_L^T \mathbf{v}_L + \mathbf{T}_E^T \dot{\mathbf{q}}_E \tag{26}$$

Proof. Item 1 is a direct consequence of (20). In order to prove items 2 and 3, let us define $\mathbf{M}(\mathbf{q}) := \mathrm{diag}[\mathbf{M}_1(\mathbf{q}_1), \mathbf{M}_2(\mathbf{q}_2), \cdots \mathbf{M}_m(\mathbf{q}_m)]$ and $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) := \mathrm{diag}[\mathbf{C}_1(\mathbf{q}_1,\dot{\mathbf{q}}_1), \mathbf{C}_2(\mathbf{q}_2,\dot{\mathbf{q}}_2), \cdots \mathbf{C}_m(\mathbf{q}_m,\dot{\mathbf{q}}_m)]$. Using (20) and (24) with the fact that $\dot{\mathbf{M}} = \mathbf{C} + \mathbf{C}^T$ (from $\dot{\mathbf{M}} - 2\mathbf{C} = -[\dot{\mathbf{M}} - 2\mathbf{C}]^T$), we then have

$$\begin{aligned} &\begin{bmatrix} \dot{\mathbf{M}}_L - 2\mathbf{C}_L & -2\mathbf{C}_{LE} \\ -2\mathbf{C}_{EL} & \dot{\mathbf{M}}_E - 2\mathbf{C}_E \end{bmatrix} \\ &= \frac{d}{dt}(\mathbf{S}^{-T}\mathbf{M}\mathbf{S}^{-1}) - 2\mathbf{S}^{-T}\mathbf{M}\frac{d}{dt}(\mathbf{S}^{-1}) - 2\mathbf{S}^{-T}\mathbf{C}\mathbf{S}^{-1} \\ &= \underbrace{\frac{d}{dt}(\mathbf{S}^{-T})\mathbf{M}\mathbf{S}^{-1} - \mathbf{S}^{-T}\mathbf{M}\frac{d}{dt}(\mathbf{S}^{-1}) - \mathbf{S}^{-T}[\mathbf{C} - \mathbf{C}^T]\mathbf{S}^{-1}}_{\text{skew-symmetric}} \end{aligned}$$

where we omit the arguments to avoid cluster. Thus, $\dot{\mathbf{M}}_L(\mathbf{q}) - 2\mathbf{C}_L(\mathbf{q},\dot{\mathbf{q}})$ and $\dot{\mathbf{M}}_E(\mathbf{q}) - 2\mathbf{C}_E(\mathbf{q},\dot{\mathbf{q}})$ are skew symmetric and $\mathbf{C}_{LE}(\mathbf{q},\dot{\mathbf{q}}) = -\mathbf{C}_{EL}^T(\mathbf{q},\dot{\mathbf{q}})$. Item 4 can also be easily shown by using (12) and (17). ■

Following Proposition 1, $\mathbf{M}_L(\mathbf{q}), \mathbf{M}_E(\mathbf{q})$ and $\mathbf{C}_L(\mathbf{q},\dot{\mathbf{q}}), \mathbf{C}_E(\mathbf{q},\dot{\mathbf{q}})$ in (22) and (23) can be thought of as inertia and Coriolis matrices of the locked and shape systems. Thus, with the cancelation of the coupling terms via the controls $\mathbf{T}_L$ and $\mathbf{T}_E$, the original mn-DOF group dynamics (10) can be decomposed into the n-DOF locked and $(m-1)n$-DOF shape systems, whose dynamics are decoupled and similar to that of the usual mechanical systems. A variety of control techniques developed for general mechanical systems can then be utilized (e.g., passivity-based control).

One remarkable property of the passive decomposition is that the decoupled system has the same passivity property as the (passive) open-loop system with the kinetic energy $\kappa(t)$ in (25) and the environmental power as the storage function and the supply rate [24], i.e., with the decoupling control $\mathbf{T}_L = \mathbf{C}_{LE}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_E$ and $\mathbf{T}_E = \mathbf{C}_{EL}(\mathbf{q},\dot{\mathbf{q}})\mathbf{v}_L$,

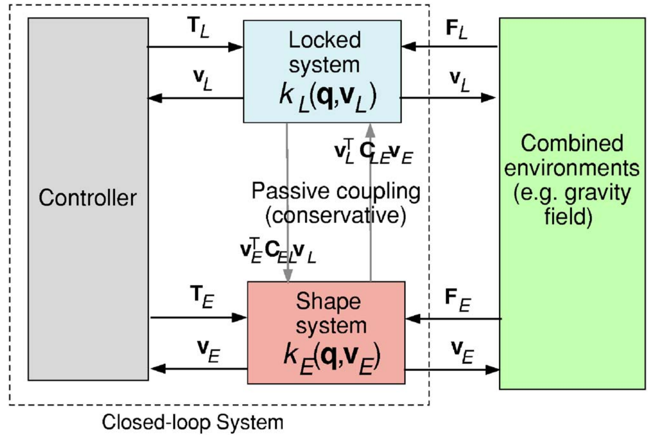

Fig. 2 A circuit-network representation of the passive decomposition, where $\kappa_L(\mathbf{q},\mathbf{v}_L) := \frac{1}{2}\mathbf{v}_L^T \mathbf{M}_L(\mathbf{q})\mathbf{v}_L$ and $\kappa_E(\dot{\mathbf{q}},\mathbf{v}_E) := \frac{1}{2}\dot{\mathbf{q}}_E^T \mathbf{M}_E(\mathbf{q})\dot{\mathbf{q}}_E$

$$\begin{aligned} \frac{d}{dt}\kappa(t) &= \underbrace{\mathbf{T}_L^T \mathbf{v}_L + \mathbf{T}_E^T \dot{\mathbf{q}}_E}_{=0 \text{ from item 3 of Proposition 1}} + \mathbf{F}_L^T \mathbf{v}_L + \mathbf{F}_E^T \dot{\mathbf{q}}_E \\ &= \underbrace{\mathbf{F}_1^T \dot{\mathbf{q}}_1 + \mathbf{F}_2^T \dot{\mathbf{q}}_2 + \ldots + \mathbf{F}_m^T \dot{\mathbf{q}}_m}_{\text{environmental power}} \end{aligned} \tag{27}$$

where the last equality is from item 4 of Proposition 1. Note that this equality (27) is also satisfied by the open-loop group dynamics (10) with $\mathbf{T}_i = \mathbf{0}$. In the orbital formation flying, with the environmental power supplied/extracted by the gravitation field, this equality (27) will determine the periodic group motion. Therefore, we would be able to decouple the formation and maneuver from each other without affecting periodicity and orbits of the group motion. This enforced passivity (27) would also enhance coupled stability of the group, when it forms a large interconnected system with other (passive) agents/groups, or it is physically coupled with (passive) external systems, as a feedback interconnection of passive systems is necessarily stable [25]. This passivity property has been successfully exploited in some robotic applications [20–23,27], and its application for multi-agent formation control is a future research topic.

A circuit network representation of the decomposed dynamics (22) and (23) is given in Fig. 2, where (1) the locked and shape systems are both passive; (2) the coupling is energetically conservative (as shown in (27)); and (3) the total group kinetic energy and control/environmental supply rates are decomposed.

In [22,23], the passive decomposition is developed in the geometric setting (i.e., coordinate invariant) for multiple mechanical systems under general holonomic constraints. Thus, using the results of [22,23], any formulations of mechanical system dynamics (e.g., w.r.t. a rotating frame or unit quaternion for motion in $SO(3)$) and holonomic constraints (e.g., distances among the agents) can be used as the group dynamics and the formation variable. However, in this paper, to make the presentation clearer with a simple decomposition (12) (and (18)), we confine our attention to the dynamics and the formation variable given by (10) and (11).

Geometrically, the locked system velocity $\mathbf{v}_L$ defines the projection of the group velocity to the level set $\mathcal{H}_{c(t)} := \{\mathbf{p} \in \mathrm{Re}^{mn} | \mathbf{q}_E(\mathbf{p}) = \mathbf{q}_E(\mathbf{q}(t))\}$, while the shape system velocity $\mathbf{v}_E$ defines its orthogonal complement w.r.t. the inertia matrix (Riemannian metric) as shown by the inertia block-diagonalizing property (20). If $\mathbf{v}_E = \mathbf{0}$ (i.e., $\mathbf{q}_E$ is constant), the shape system dynamics and the coupling term $\mathbf{C}_{LE}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_E$ will vanish, and the locked system dynamics and the coupling term $\mathbf{C}_{EL}(\mathbf{q},\dot{\mathbf{q}})\mathbf{v}_L$ will become the constrained dynamics (Levi-Civita connection) on

$\mathcal{H}_{c(t)}$ and the second fundamental form, respectively [33]. For more details on the geometric property of the passive decomposition, refer to [22,23].

4 Centralized Maneuver and Formation Control Design

Once the group dynamics is decomposed as in (22) and (23), design of formation and maneuver controls becomes fairly straightforward, as the dynamics of the shape system (i.e., formation) and the locked system (maneuver) are similar to that of usual mechanical systems. These controls, which will be designed in this section, however, generally require the state information of all group agents (e.g., via centralized communication). In Sec. 5, for applications where such communication requirement is not achievable, we will provide a controller decentralization which only requires undirected line communication graph topology among the agents. See Sec. 7.1 for a detailed procedure of applying the centralized control to a specific example (formation flying of three rigid bodies).

4.1 Control Design for Group Translation. From (16), the locked system velocity of the group translation is given by

$$\mathbf{v}_L = \frac{m_1\dot{\mathbf{x}}_1 + m_2\dot{\mathbf{x}}_2 + \cdots + m_m\dot{\mathbf{x}}_n}{m_1 + m_2 + \cdots + m_m} \in \mathrm{Re}^3 \tag{28}$$

thus, we can define the translation locked system configuration by

$$\mathbf{x}_L := \frac{m_1\mathbf{x}_1 + m_2\mathbf{x}_2 + \cdots + m_m\mathbf{x}_m}{m_1 + m_2 + \cdots + m_m} \in \mathrm{Re}^3 \tag{29}$$

i.e., the position of the center of mass. We choose this $\mathbf{x}_L$ as the maneuver variable for the group translation so that the maneuver and formation aspects are decoupled from each other. With this map $\mathbf{x}_L : \mathrm{Re}^{3m} \to \mathrm{Re}^3$ (29), we can project the translation locked system dynamics on Re^3. As shown in [22,23], this projection is possible since the manifold of the group translation dynamics (1) is flat (i.e., Riemannian curvature vanishes [31] as shown by the constant mass m_i on its flat configuration space Re^3), and the level sets of the formation variable (8) are also given by flat planes. For more details, refer to [22,23].

Following (22) and (23) with the definition of $\mathbf{x}_L$ in (29), the group translation dynamics (1) is decomposed s.t.

$$\mathbf{M}_L\ddot{\mathbf{x}}_L = \mathbf{t}_L + \mathbf{f}_L \tag{30}$$

$$\mathbf{M}_E\ddot{\mathbf{x}}_E = \mathbf{t}_E + \mathbf{f}_E \tag{31}$$

where $\mathbf{x}_L \in \mathrm{Re}^3$ and $\mathbf{x}_E \in \mathrm{Re}^{3(m-1)}$ are the locked and shape systems configurations given in (29) and (8), $\mathbf{M}_L = (m_1 + m_2 + \cdots + m_m)\mathbf{I}_{3\times 3}$ and $\mathbf{M}_E \in \mathrm{Re}^{3(m-1)\times 3(m-1)}$ are the symmetric and positive definite mass matrices, $\mathbf{t}_L \in \mathrm{Re}^3$ and $\mathbf{t}_E \in \mathrm{Re}^{3(m-1)}$ are the controls (to be designed), and $\mathbf{f}_L = \mathbf{f}_1 + \mathbf{f}_2 + \cdots + \mathbf{f}_m \in \mathrm{Re}^3$, $\mathbf{f}_E \in \mathrm{Re}^{3(m-1)}$ are the environmental disturbances on the translation locked and shape systems, respectively.

To achieve the control objectives (5) and (6), we design the locked and shape system controls $\mathbf{t}_L, \mathbf{t}_E$ to be

$$\mathbf{t}_L := \mathbf{M}_L\ddot{\mathbf{x}}_L^d(t) - \mathbf{K}_v^L[\dot{\mathbf{x}}_L - \dot{\mathbf{x}}_L^d(t)] - \mathbf{K}_p^L[\mathbf{x}_L - \mathbf{x}_L^d(t)] \tag{32}$$

$$\mathbf{t}_E := \mathbf{M}_E\ddot{\mathbf{x}}_E^d(t) - \mathbf{K}_v^E[\dot{\mathbf{x}}_E - \dot{\mathbf{x}}_E^d(t)] - \mathbf{K}_p^E[\mathbf{x}_E - \mathbf{x}_E^d(t)] \tag{33}$$

where $\mathbf{x}_L^d(t) \in \mathrm{Re}^3$, $\mathbf{x}_E^d(t) \in \mathrm{Re}^{3(m-1)}$ are the desired translation maneuver and formation trajectories, $\mathbf{K}_v^L, \mathbf{K}_p^L \in \mathrm{Re}^{3\times 3}$ and $\mathbf{K}_v^E, \mathbf{K}_p^E \in \mathrm{Re}^{3(m-1)\times 3(m-1)}$ are symmetric and positive-definite proportional-derivative (PD) control gains. The environmental disturbance $\mathbf{f}_i$ in (1) can be compensated for either by individual local controllers or by incorporating cancelation of $\mathbf{f}_L, \mathbf{f}_E$ of (30) and (31) into the controls (32) and (33).

4.2 Control Design for Group Attitude. Following (22) and (23), the group attitude dynamics (2) with the formation variable $\boldsymbol{\zeta}_E(t) \in \mathrm{Re}^{3(m-1)}$ (9) is decomposed s.t.

$$\mathbf{H}_L(\boldsymbol{\zeta})\dot{\boldsymbol{\omega}}_L + \mathbf{Q}_L(\boldsymbol{\zeta},\boldsymbol{\omega})\boldsymbol{\omega}_L + \mathbf{Q}_{LE}(\boldsymbol{\zeta},\boldsymbol{\omega})\dot{\boldsymbol{\zeta}}_E = \boldsymbol{\tau}_L + \boldsymbol{\delta}_L \tag{34}$$

$$\mathbf{H}_E(\boldsymbol{\zeta})\ddot{\boldsymbol{\zeta}}_E + \mathbf{Q}_E(\boldsymbol{\zeta},\boldsymbol{\omega})\dot{\boldsymbol{\zeta}}_E + \mathbf{Q}_{EL}(\boldsymbol{\zeta},\boldsymbol{\omega})\boldsymbol{\omega}_L = \boldsymbol{\tau}_E + \boldsymbol{\delta}_E \tag{35}$$

where each term is defined according to its counterparts in (22) and (23) with $\boldsymbol{\zeta} := [\boldsymbol{\zeta}_1^T, \boldsymbol{\zeta}_2^T, \ldots, \boldsymbol{\zeta}_m^T]^T \in \mathrm{Re}^{3m}$ and $\boldsymbol{\omega}(t) := d/dt\boldsymbol{\zeta}(t) \in \mathrm{Re}^{3m}$. Here, from (16), the locked system velocity $\boldsymbol{\omega}_L(t) \in \mathrm{Re}^3$ is given by

$$\boldsymbol{\omega}_L := \left[\sum_{j=1}^{m}\mathbf{H}_j(\boldsymbol{\zeta}_j)\right]^{-1}[\mathbf{H}_1(\boldsymbol{\zeta}_1)\boldsymbol{\omega}_1 + \mathbf{H}_2(\boldsymbol{\zeta}_2)\boldsymbol{\omega}_2 + \cdots + \mathbf{H}_m(\boldsymbol{\zeta}_m)\boldsymbol{\omega}_m] \tag{36}$$

In contrast to the translational locked system (30), as shown in [22], the attitude locked system (34) does not have a well-defined configuration on Re^3 in the sense that there does not exist a function $\boldsymbol{\zeta}_L(t) \in \mathrm{Re}^3$ s.t. $(d/dt)\boldsymbol{\zeta}_L = \boldsymbol{\omega}_L$ (i.e., $\boldsymbol{\omega}_L$ is a pseudo-velocity [34]). Therefore, the maneuver control objective described by a (position) trajectory $\boldsymbol{\zeta}_L^d(t) \in \mathrm{Re}^3$ would not be achievable by the locked system (34). The lack of this projection is due to the fact that the distribution, which is orthogonal to the level sets of the formation variable (9) w.r.t. the inertia metric is not integrable. For more detail, see [22,23].

This maneuver trajectory tracking, however, can still be achieved if the group dynamics (2) is confined in a single level set of the formation variable (9) and the desired maneuver trajectory $\boldsymbol{\zeta}_L^d(t)$ is defined on this level set. Thus, when the position of the locked system (34) needs to be controlled, we restrict the desired formation $\boldsymbol{\zeta}_E^d$ to be constant and define $\boldsymbol{\zeta}_L^d(t)$ on the target level set $\mathcal{H}_d := \{(\boldsymbol{\zeta}_1, \boldsymbol{\zeta}_2, \ldots, \boldsymbol{\zeta}_m) \in \mathrm{Re}^{3m} | \boldsymbol{\zeta}_E(\boldsymbol{\zeta}_1, \boldsymbol{\zeta}_2, \ldots, \boldsymbol{\zeta}_m) = \boldsymbol{\zeta}_E^d\}$. Since the relative attitude between m-agents is rigidly fixed on $\mathcal{H}_d$ (i.e., $\boldsymbol{\zeta}_E$ is constant), the total group maneuver on $\mathcal{H}_d$ can be specified by the attitude of *any* agent on $\mathcal{H}_d$. Due to this property, we can design $\boldsymbol{\zeta}_L^d(t)$ as a desired trajectory for any one of the agents. For example, if the first agent is chosen, $\boldsymbol{\zeta}_L^d(t)$ will be designed s.t. the desired maneuver achieved by $\boldsymbol{\zeta}_1(t) \to \boldsymbol{\zeta}_L^d(t)$ on $\mathcal{H}_d$.

While the desired maneuver trajectory $\boldsymbol{\zeta}_L^d(t)$ is defined on $\mathcal{H}_d$, the control for it must operate even when $\boldsymbol{\zeta}_E(t) = \boldsymbol{\zeta}_E^d$ is not perfectly achieved. To do this, we define the following map $\tilde{\boldsymbol{\zeta}}_L : (\boldsymbol{\zeta}_1, \boldsymbol{\zeta}_2, \cdots \boldsymbol{\zeta}_m) \mapsto \mathrm{Re}^3$ s.t.

$$\tilde{\boldsymbol{\zeta}}_L(t) := \mathbf{A}_1\boldsymbol{\zeta}_1(t) + \mathbf{A}_2\boldsymbol{\zeta}_2(t)\cdots + \mathbf{A}_m\boldsymbol{\zeta}_m(t) + \mathbf{b} \in \mathrm{Re}^3 \tag{37}$$

where $\mathbf{A}_k \in \mathrm{Re}^{3\times 3}$ are full-rank matrices s.t. $\Sigma_{k=1}^m \mathbf{A}_k = \mathbf{I}$ to ensure $(d/dt)\tilde{\boldsymbol{\zeta}}_L = \boldsymbol{\omega}_L$, when $(\boldsymbol{\zeta}_1, \boldsymbol{\zeta}_2, \ldots, \boldsymbol{\zeta}_m) \in \boldsymbol{\mathcal{H}}_d$ (i.e., on $\mathcal{H}_d$ where $\boldsymbol{\omega}_1 = \boldsymbol{\omega}_2 \cdots = \boldsymbol{\omega}_m$, from (36) and (37) with $\Sigma_{k=1}^m \mathbf{A}_k = \mathbf{I}$, $(d/dt)\tilde{\boldsymbol{\zeta}}_L = \boldsymbol{\omega}_L = \boldsymbol{\omega}_i$, $\forall i \in \{1, \ldots, m\}$). Also, the vector $\mathbf{b} \in \mathrm{Re}^3$ in (37) is designed to enforce consistency between the map $\tilde{\boldsymbol{\zeta}}_L$ and the designed trajectory $\boldsymbol{\zeta}_L^d(t)$ on $\mathcal{H}_d$ (e.g., if $\boldsymbol{\zeta}_L^d(t)$ is designed for the first agent, $\mathbf{b}$ will be chosen s.t. $\tilde{\boldsymbol{\zeta}}_L(t) = \boldsymbol{\zeta}_1(t)$ when $(\boldsymbol{\zeta}_i, \boldsymbol{\zeta}_2, \ldots, \boldsymbol{\zeta}_m) \in \mathcal{H}_d$). We choose this map $\tilde{\boldsymbol{\zeta}}_L(t)$ (37) as the attitude maneuver variable. We can then rewrite the maneuver control objective (5) for the attitude dynamics s.t.

$$\tilde{\boldsymbol{\zeta}}_L(t) \to \boldsymbol{\zeta}_L^d(t) \quad \text{on } \mathcal{H}_d \tag{38}$$

Geometrically, with this map $\tilde{\boldsymbol{\zeta}}_L$ in (37), the desired maneuver $\boldsymbol{\zeta}_L^d(t)$ is *lifted* from $\mathcal{H}_d$ to the ambient configuration space (Re^{3m}) of the group attitude dynamics (2). Therefore, control action can be defined in the ambient manifold for this lifted desired maneuver (see (39)). Since the map $\tilde{\boldsymbol{\zeta}}_L$ is smooth, this defined control will smoothly converge to the desired one on $\mathcal{H}_d$ as $\boldsymbol{\zeta}_E(t) \to \boldsymbol{\zeta}_E^d$.

Using the maneuver variable $\tilde{\boldsymbol{\zeta}}_L(t)$ in (37) as a "pseudoconfiguration" for the locked system (34), we design the attitude locked and shape system controls to be

$$\tau_L = \mathbf{Q}_{LE}(\boldsymbol{\zeta},\boldsymbol{\omega})\dot{\boldsymbol{\zeta}}_E(t) + \mathbf{H}_L(\boldsymbol{\zeta})\ddot{\boldsymbol{\zeta}}_L^d(t) + \mathbf{Q}_L(\boldsymbol{\zeta},\boldsymbol{\omega})\dot{\boldsymbol{\zeta}}_L^d(t) - \boldsymbol{\lambda}_v^L(\boldsymbol{\omega}_L(t) - \dot{\boldsymbol{\zeta}}_L^d(t)) - \boldsymbol{\lambda}_p^L(\tilde{\boldsymbol{\zeta}}_L(t) - \boldsymbol{\zeta}_L^d(t)) \tag{39}$$

$$\tau_E = \mathbf{Q}_{EL}(\boldsymbol{\zeta},\boldsymbol{\omega})\boldsymbol{\omega}_L(t) + \mathbf{H}_E(\boldsymbol{\zeta})\ddot{\boldsymbol{\zeta}}_E^d(t) + \mathbf{Q}_E(\boldsymbol{\zeta},\boldsymbol{\omega})\dot{\boldsymbol{\zeta}}_E^d(t) - \boldsymbol{\lambda}_v^E(\dot{\boldsymbol{\zeta}}_E(t) - \dot{\boldsymbol{\zeta}}_E^d(t)) - \boldsymbol{\lambda}_p^E(\boldsymbol{\zeta}_E(t) - \boldsymbol{\zeta}_E^d(t)) \tag{40}$$

where $\mathbf{Q}_{LE}(\boldsymbol{\zeta},\boldsymbol{\omega})\dot{\boldsymbol{\zeta}}_E$ and $\mathbf{Q}_{EL}(\boldsymbol{\zeta},\boldsymbol{\omega})\boldsymbol{\omega}_L$ are the decoupling controls, $\boldsymbol{\zeta}_E^d(t) \in \mathrm{Re}^{3(m-1)}$ is the desired formation trajectory, $\boldsymbol{\zeta}_L^d(t) \in \mathrm{Re}^3$ is the desired maneuver profile defined on the target level set $\mathcal{H}_d$, and $\boldsymbol{\lambda}_v^L, \boldsymbol{\lambda}_p^L \in \mathrm{Re}^{3\times 3}$ and $\boldsymbol{\lambda}_v^E, \boldsymbol{\lambda}_p^E \in \mathrm{Re}^{3(m-1)\times 3(m-1)}$ are symmetric and positive-definite PD-control gains. The environmental disturbances $\boldsymbol{\delta}_i$ in (2) can be compensated for either by individual local controllers or by incorporating cancellation of $\boldsymbol{\delta}_L, \boldsymbol{\delta}_E$ of (34) and (35) into the attitude locked and shape controls (39) and (40).

When the maneuver tracking (38) is desired, we can utilize large PD-control gains $\boldsymbol{\lambda}_v^E, \boldsymbol{\lambda}_p^E$ in (40) (with a constant $\boldsymbol{\zeta}_E^d$) so that the group attitude quickly converges to $\mathcal{H}_d$, and, from there, the locked control (39) will enforce the desired maneuver as specified by (38). The desired formation $\boldsymbol{\zeta}_E^d(t)$ can still be time-varying when we do not need to explicitly control the position of the locked system (e.g., velocity tracking $\boldsymbol{\omega}_L(t) \to \boldsymbol{\omega}_L^d(t)$). The properties of the (stabilizing) centralized controls (32), (33), (39), and (40) are summarized in the following theorem.

THEOREM 1.

(1) *Consider the group translation dynamics* (1) *under the centralized control* (32) and (33). *Suppose that the disturbances* $\mathbf{f}_i$ *are negligible (e.g., by local cancelation). Then,* $(\dot{\mathbf{x}}_L(t) - \dot{\mathbf{x}}_L^d(t), \mathbf{x}_L(t) - \mathbf{x}_L^d(t)) \to \mathbf{0}$ *and* $(\dot{\mathbf{x}}_E(t) - \dot{\mathbf{x}}_E^d(t), \mathbf{x}_E(t) - \mathbf{x}_E^d(t)) \to \mathbf{0}$ *exponentially.*

(2) *Consider the group attitude dynamics* (2) *under the centralized control* (39) and (40). *Suppose that the inertia matrix* $\mathbf{H}_i(\boldsymbol{\zeta}_i)$ *and the Coriolis matrix* $\mathbf{Q}_i(\boldsymbol{\zeta}_i,\boldsymbol{\omega}_i)$ *are bounded. Suppose further that the disturbances* $\boldsymbol{\delta}_i$ *are negligible (e.g., by local cancelation) and the target formation* $\boldsymbol{\zeta}_E^d$ *is constant. Then,* $(\dot{\tilde{\boldsymbol{\zeta}}}_L(t) - \dot{\boldsymbol{\zeta}}_L^d(t), \tilde{\boldsymbol{\zeta}}_L(t) - \boldsymbol{\zeta}_L^d(t)) \to \mathbf{0}$ *and* $(\dot{\boldsymbol{\zeta}}_E(t), \boldsymbol{\zeta}_E(t) - \boldsymbol{\zeta}_E^d) \to \mathbf{0}$ *exponentially*.

Proof.

(1) This is a direct consequence of the fact that the translation locked and shape systems (30) and (31) under the centralized control (32) and (33) are given by the linear time invariant (LTI) mass-spring-damper dynamics.

(2) With a constant $\boldsymbol{\zeta}_E^d$, the shape system dynamics (35) under the centralized control (40) is given by:

$$\mathbf{H}_E(\boldsymbol{\zeta})\ddot{\boldsymbol{\zeta}}_E + \mathbf{Q}_E(\boldsymbol{\zeta},\boldsymbol{\omega})\dot{\boldsymbol{\zeta}}_E - \boldsymbol{\lambda}_v^E\dot{\boldsymbol{\zeta}}_E(t) - \boldsymbol{\lambda}_p^E(\boldsymbol{\zeta}_E(t) - \boldsymbol{\zeta}_E^d(t)) = \boldsymbol{\delta}_E \tag{41}$$

The exponential convergence of $(\dot{\boldsymbol{\zeta}}_E, \boldsymbol{\zeta}_E - \boldsymbol{\zeta}_E^d) \to \mathbf{0}$ with bounded $\mathbf{H}_E(\boldsymbol{\zeta})$ and $\mathbf{Q}_E(\boldsymbol{\zeta},\boldsymbol{\omega})$ and $\boldsymbol{\delta}_i = \mathbf{0}$ can then be proved by using the Lyapunov function $V_\zeta(t) = \frac{1}{2}\dot{\boldsymbol{\zeta}}_E^T\mathbf{H}_E\dot{\boldsymbol{\zeta}}_E + \frac{1}{2}\boldsymbol{\zeta}_E^T\boldsymbol{\lambda}_p^E\boldsymbol{\zeta}_E + \epsilon\boldsymbol{\zeta}_E^T\mathbf{H}_E\dot{\boldsymbol{\zeta}}_E$, where $\epsilon > 0$ is a small scalar s.t. $V_\zeta(t)$ is positive definite. For more details, refer to p. 194 of [35].

Let us denote the exponential convergence rate of (41) by $\gamma > 0$. Since $\boldsymbol{\zeta}_E \to \mathbf{0}$ exponentially, from the definition (9), there then exists a finite scalar $\alpha > 0$ s.t. for all $i, j = 1, \ldots, m$:

$$|\boldsymbol{\omega}_i^k(t) - \boldsymbol{\omega}_j^k(t)| \leq \alpha e^{-\gamma t} \tag{42}$$

where $\boldsymbol{\omega}_i^k(t)$ is the kth component of $\boldsymbol{\omega}_i(t) \in \mathrm{Re}^3$. Thus, using (42) with the definitions of $\boldsymbol{\omega}_L$ and $\tilde{\boldsymbol{\zeta}}_L$ in (36) and (37) and the boundedness of $\mathbf{A}_i$ and $\mathbf{H}_i(\boldsymbol{\zeta}_i)$, we can show that there exists a finite scalar $\beta > 0$ s.t. (with arguments omitted)

$$\begin{aligned} |\dot{\tilde{\zeta}}_L^k - \boldsymbol{\omega}_L^k| &= |(\mathbf{A}_1^k - \phi_1^k)\boldsymbol{\omega}_1 + \cdots (\mathbf{A}_m^k - \phi_m^k)\boldsymbol{\omega}_m| \\ &\leq \left| \underbrace{\left(\sum_{j=1}^m (\mathbf{A}_j^k - \phi_j^k) \right)}_{=\mathbf{0}} \boldsymbol{\omega}_i \right| + \beta e^{-\gamma t} \end{aligned} \tag{43}$$

where i is any integer in $\{1, \ldots, m\}$, and $\mathbf{A}_j^k$ and $\phi_j^k(\boldsymbol{\zeta})$ are the kth row vector ($k = 1, \ldots, 3(m-1)$) of $\mathbf{A}_j$ and $\phi_j(\boldsymbol{\zeta}) := \mathbf{H}_L^{-1}(\boldsymbol{\zeta})\mathbf{H}_j(\boldsymbol{\zeta}_j)$. Here $[\sum_{j=1}^m (\mathbf{A}_j^k - \phi_j^k(\boldsymbol{\zeta}))] = \mathbf{0}$, since $\sum_{j=1}^m \mathbf{A}_j = \sum_{j=1}^m \phi_j(\boldsymbol{\zeta}) = \mathbf{I}$.

From (41) with $\boldsymbol{\delta}_E = \mathbf{0}$, we can also show that $\ddot{\boldsymbol{\zeta}}_E(t) \to \mathbf{0}$ exponentially, since $(\dot{\boldsymbol{\zeta}}_E, \boldsymbol{\zeta}_E - \boldsymbol{\zeta}_E^d) \to \mathbf{0}$ exponentially, $\mathbf{Q}_L(\boldsymbol{\zeta},\dot{\boldsymbol{\zeta}})$ is bounded, and $\mathbf{H}_L(\boldsymbol{\zeta})$ is bounded and positive definite. Thus, similar to (42), there exists a finite scalar $\bar{\alpha} > 0$ s.t. $|\dot{\boldsymbol{\omega}}_i^k(t) - \dot{\boldsymbol{\omega}}_j^k(t)| \leq \bar{\alpha}e^{-\gamma t}$ for any $i, j = 1, \ldots, m$. Therefore, similar to (43), we can find a finite scalar $\bar{\beta} > 0$ s.t. for any $i \in \{1, \ldots, m\}$:

$$\begin{aligned} |\ddot{\tilde{\zeta}}_L^k - \dot{\boldsymbol{\omega}}_L^k| &= \left| \sum_{j=1}^m (\mathbf{A}_j^k - \phi_j^k)\dot{\boldsymbol{\omega}}_j - \dot{\phi}_j^k\boldsymbol{\omega}_j \right| \\ &\leq \left| \underbrace{\left(\sum_{j=1}^m (\mathbf{A}_j^k - \phi_j^k) \right)}_{=\mathbf{0}} \dot{\boldsymbol{\omega}}_i \right| + \left| \underbrace{\left(\sum_{j=1}^m \dot{\phi}_j^k \right)}_{=\mathbf{0}} \boldsymbol{\omega}_i \right| + \bar{\beta}e^{-\gamma t} \end{aligned} \tag{44}$$

where we use the fact that (1) $\forall i, j \in \{1, \ldots, m\}$, $\boldsymbol{\omega}_i - \boldsymbol{\omega}_j \to 0$, exponentially (from (42)); (2) $\sum_{j=1}^m \dot{\phi}_j = \mathbf{0}$, as $\sum_{j=1}^m \phi_j = \mathbf{I}$; and (3) $\dot{\phi}_j(\boldsymbol{\zeta}) = \mathbf{H}_L^{-1}\dot{\mathbf{H}}_j - \mathbf{H}_L^{-1}\dot{\mathbf{H}}_L\mathbf{H}_L^{-1}\mathbf{H}_j$ is bounded, since $\dot{\mathbf{H}}_\star = \mathbf{Q}_\star + \mathbf{Q}_\star^T$ (from $\dot{\mathbf{H}}_\star - \mathbf{Q}_\star = -[\dot{\mathbf{H}}_\star - \mathbf{Q}_\star]^T$) is bounded with bounded $\mathbf{Q}_\star$ ($\star \in \{1, \ldots, m, L\}$).

Thus, with (43) and (44) and the bounded $\mathbf{H}_\star, \mathbf{Q}_\star$ ($\star \in \{1, \ldots, m, L\}$), the locked system dynamics (34) under the control (39) can be written as:

$$\mathbf{H}_L(\boldsymbol{\zeta})(\ddot{\tilde{\boldsymbol{\zeta}}}_L - \ddot{\boldsymbol{\zeta}}_L^d) + \mathbf{Q}_L(\boldsymbol{\zeta},\boldsymbol{\omega})(\dot{\tilde{\boldsymbol{\zeta}}}_L - \dot{\boldsymbol{\zeta}}_L^d) + \boldsymbol{\lambda}_v^L(\dot{\tilde{\boldsymbol{\zeta}}}_L - \dot{\boldsymbol{\zeta}}_L^d) + \boldsymbol{\lambda}_p^L(\tilde{\boldsymbol{\zeta}}_L - \boldsymbol{\zeta}_L^d) = \boldsymbol{\delta}_L + \mathbf{d}(t) \tag{45}$$

where $\mathbf{d}(t) \in \mathrm{Re}^3$ is an exponentially decaying vector s.t. $|\mathbf{d}(t)| \leq De^{\gamma t}$ with a finite scalar $D \geq 0$. Thus, if $\boldsymbol{\delta}_i = \mathbf{0}$, the locked system dynamics (45) is the exponentially stable dynamics [35] with the exponentially decaying disturbance $\mathbf{d}(t)$. Thus, $(\dot{\tilde{\boldsymbol{\zeta}}}_L - \dot{\boldsymbol{\zeta}}_L^d, \tilde{\boldsymbol{\zeta}}_L - \boldsymbol{\zeta}_L^d) \to \mathbf{0}$ exponentially, with the converging speed determined by either the exponential rate of the left-hand side of (41) (i.e., γ) or that of (45), whichever is slower. ■

The boundedness assumption on $\mathbf{H}_i(\boldsymbol{\zeta}_i)$ and $\mathbf{Q}_i(\boldsymbol{\zeta}_i,\boldsymbol{\omega}_i)$ in item 2 of Theorem 1 can be achieved if the desired maneuver velocity $(d/dt)\boldsymbol{\zeta}_L^d(t)$ and the initial angular rates $\boldsymbol{\omega}_i(0)$ ($i = 1, \ldots, m$) are bounded. This is because (1) for the usual rigid-body attitude dynamics (2), the inertia matrix $\mathbf{H}_i(\boldsymbol{\zeta}_i)$ is bounded w.r.t $\boldsymbol{\zeta}_i$, and the Coriolis matrix $\mathbf{Q}_i(\boldsymbol{\zeta}_i,\boldsymbol{\omega}_i)$ is bounded w.r.t. $\boldsymbol{\zeta}_i$ and linear w.r.t. $\boldsymbol{\omega}_i$ [36]; (2) the configuration space of the attitude dynamics (2) is bounded, thus, with $\boldsymbol{\zeta}_i$ being bounded, $\mathbf{H}_i(\boldsymbol{\zeta}_i)$ is bounded; and (3) from item 2 of Theorem 1, once $(d/dt)\boldsymbol{\zeta}_L^d(t)$ and $\boldsymbol{\omega}_i(0)$ are bounded, $\boldsymbol{\omega}_i(t)$ will be bounded, thus, with bounded $\boldsymbol{\zeta}_i$, $\mathbf{Q}_i(\boldsymbol{\zeta}_i,\boldsymbol{\omega}_i)$ will also be bounded.

Note from Theorem 1 that, under the controls (32), (33), (39), and (40), the translation and attitude of each agent (1) and (2) are converging exponentially to the desired position and attitude.

Therefore, this stability would still be preserved under some small perturbation (e.g., sensor and actuator noises, model uncertainty, etc.), and the trajectory of each agent will be close to the desired one, as long as this perturbation is small enough.

4.3 Hierarchy by Abstraction. Since the locked systems (30) and (34) have the dynamics similar to those of the individual agent ((1) and (2)), abstracting each group by its locked system, we can generate a hierarchy among multiple agents/groups in a very natural way as follows. Consider N agents. From these N agents, let us designate q-agents and, among the rest, make p-groups ${}^{o}\Sigma_j (j=1,\ldots,p)$, each consisting of m_j agents (i.e., $N = q+\Sigma_{j=1}^{p} m_j$). Suppose that we want to control the formation among these q-agents and p-groups, each considered as one entity.

For the translation, following (30) and (31), we can then decompose each group ${}^{o}\Sigma_j$ into its (1) locked system ${}^{1}L_j$: ${}^{1j}\mathbf{M}_L {}^{1j}\ddot{\mathbf{x}}_L = {}^{1j}\mathbf{t}_L + {}^{1j}\mathbf{f}_L$; and (2) shape system ${}^{1}S_j$: ${}^{1j}\mathbf{M}_E {}^{1j}\ddot{\mathbf{x}}_E = {}^{1j}\mathbf{t}_E + {}^{1j}\mathbf{f}_E$, where ${}^{1j}\mathbf{x}_L \in \mathrm{Re}^3$ and ${}^{1j}\mathbf{x}_E \in \mathrm{Re}^{3(m_j-1)}$. Here, we use notation ${}^{1j}\star$ in the place of $\star$ in (30) and (31) to show that ${}^{j}L_j$ (and ${}^{j}S_j$) is one level higher than their group agents. Similarly, each agent's dynamics in ${}^{o}\Sigma_j$ may be written as ${}^{oj}m_k {}^{oj}\ddot{\mathbf{x}}_k = {}^{oj}\mathbf{t}_k + {}^{oj}\mathbf{f}_k$ $(k=1,\ldots,m_j)$ to show that they are a level lower. Then, a desired formation in the group ${}^{o}\Sigma_j$ can be achieved by controlling ${}^{1}S_j$. Since the dynamics of each ${}^{1}L_j$ has the same structure as that of a single agent (1), we can also make another group ${}^{1}\Sigma$ by collecting the q-agents and the p-locked systems ${}^{1}L_j, {}^{1}L_j, \ldots, {}^{1}L_p$ and we can define the locked and shape systems ${}^{2}L$ and ${}^{2}S$ for this group ${}^{1}\Sigma$. By controlling ${}^{2}S$, we can then control the formation shape among the q-agents and the p-groups ${}^{o}\Sigma_1, {}^{o}\Sigma_2, \ldots, {}^{o}\Sigma_p$, whereas, by controlling ${}^{2}L$, we can control the overall behavior of the total collection of the q-agents and the p-groups. We can also put this locked system ${}^{2}L$ into another group with other agents/groups. By doing so, we can generate a hierarchy as shown in Fig. 1.

A similar procedure can also be obtained for the attitude case, if each shape system (35) converges to its (constant) set point $\boldsymbol{\zeta}_E^d$ much faster (e.g., with large gains $\boldsymbol{\lambda}_v^E, \boldsymbol{\lambda}_p^E$) than its locked system dynamics (34), so that, as stated after (37), its locked system (34) will have the same structure as that of a single agent (2) with a well-defined configuration $\tilde{\boldsymbol{\zeta}}_L$ (i.e., $(d/dt)\tilde{\boldsymbol{\zeta}}_L = \boldsymbol{\omega}_L$ on $\mathcal{H}_d$). Just as in the translation case, treating the locked system as a single agent, we can then make a mixed group consisting of agents and locked systems, thus, we can generate the hierarchy in Fig. 1. This concept of hierarchy by abstraction is useful for controlling a mixed collection of many agents and groups. For example, see Sec. 7.3.

5 Controller Decentralization

As shown below, the centralized controls ((32), (33), (39), and (40)) require state information of all group agents. This communication requirement may be too expensive or infeasible for some applications. In this section, we provide a controller decentralization that only requires each agent to communicate with (or sense of) its two (or one depending on the agent numbering) neighbors. With the help of the passive decomposition, we also show that the performance of this decentralized controller would still be satisfactory, if the desired group behavior is slow.

We first choose the gain matrices $\mathbf{K}_v^E$ and $\mathbf{K}_p^E$ in (33) to be block diagonal s.t. $\mathbf{K}_\star^E = \mathrm{diag}[\mathbf{K}_\star^{E1}, \mathbf{K}_\star^{E2}, \ldots, \mathbf{K}_\star^{E(m-1)}]$ $(\star = \{v, p\})$ where $\mathbf{K}_\star^{Ej} \in \mathrm{Re}^{3\times 3}$ $(j=1,\ldots,m-1)$ are symmetric and positive-definite matrices. This is always possible, since, in (33), the only condition imposed on $\mathbf{K}_v^E$ and $\mathbf{K}_p^E$ is that they are symmetric and positive definite. Using (17) with the definition of $\mathbf{S}(\mathbf{q})$ in (12), the centralized translation controls (32) and (33) can then be decoded into the individual control $\mathbf{t}_i \in \mathrm{Re}^3$ of the ith agent (1) s.t. for $i = 1, \ldots, m$,

$$\begin{aligned}\mathbf{t}_i = {} & \underbrace{m_i \ddot{\mathbf{x}}_L^d}_{\text{decentralizable}} - \frac{m_i}{m_1 + m_2 + \ldots + m_m}[\mathbf{K}_v^L(\dot{\mathbf{x}}_L - \dot{\mathbf{x}}_L^d) + \mathbf{K}_p^L(\mathbf{x}_L - \mathbf{x}_L^d)] \\ & - \underbrace{\mathbf{M}_E^{i-1}\ddot{\mathbf{x}}_E^d + \mathbf{K}_v^{E(i-1)}(\dot{\mathbf{x}}_{i-1} - \dot{\mathbf{x}}_i - \dot{\mathbf{x}}_{E(i-1)}^d) + \mathbf{K}_p^{E(i-1)}(\mathbf{x}_{i-1} - \mathbf{x}_i - \mathbf{x}_{E(i-1)}^d)}_{\text{decentralizable}} \\ & + \underbrace{\mathbf{M}_E^{i}\ddot{\mathbf{x}}_E^d - \mathbf{K}_v^{E(i)}(\dot{\mathbf{x}}_i - \dot{\mathbf{x}}_{i+1} - \dot{\mathbf{x}}_{E(i)}^d) - \mathbf{K}_p^{E(i)}(\mathbf{x}_i - \mathbf{x}_{i+1} - \mathbf{x}_{E(i)}^d)}_{\text{decentralizable}}\end{aligned} \tag{46}$$

where $\mathbf{x}_{Ej}^d \in \mathrm{Re}^3$ is the jth entity of $\mathbf{x}_E$ in (8) s.t. $\mathbf{x}_E^d = [\mathbf{x}_{E1}^{dT}, \mathbf{x}_{E2}^{dT}, \ldots, \mathbf{x}_{E(m-1)}^{dT}]^T \in \mathrm{Re}^{3(m-1)}$, and $\mathbf{M}_E^j \in \mathrm{Re}^{3\times 3(m-1)}$ is the horizontal partition of the (constant) shape system mass matrix $\mathbf{M}_E \in \mathrm{Re}^{3(m-1)\times 3(m-1)}$ in (31) s.t. $\mathbf{M}_E = [\mathbf{M}_E^{1T}, \mathbf{M}_E^{2T}, \ldots, \mathbf{M}_E^{(m-1)T}]$. In (46), any terms having an index less than 1 or larger than m are to be zeros.

Similar to (46), with block-diagonal gain matrices $\boldsymbol{\lambda}_v^E, \boldsymbol{\lambda}_p^E$ for the shape system control (40), the centralized attitude controls (39) and (40) are decoded into individual control action s.t.

$$\begin{aligned}\boldsymbol{\tau}_i = {} & \mathbf{H}_i \mathbf{H}_L^{-1}\mathbf{Q}_{LE}\dot{\boldsymbol{\zeta}}_E + \underbrace{\mathbf{H}_i \ddot{\boldsymbol{\zeta}}_L^d}_{\text{decentralizable}} + \mathbf{H}_i \mathbf{H}_L^{-1}[\mathbf{Q}_L \dot{\boldsymbol{\zeta}}_L^d - \boldsymbol{\lambda}_v^L(\boldsymbol{\omega}_L - \dot{\boldsymbol{\zeta}}_L^d) - \boldsymbol{\lambda}_p^L(\boldsymbol{\zeta}_L - \boldsymbol{\zeta}_L^d)] \\ & - \mathbf{Q}_{EL}^{i-1}\boldsymbol{\omega}_L - \mathbf{H}_E^{i-1}\ddot{\boldsymbol{\zeta}}_E^d - \mathbf{Q}_E^{i-1}\dot{\boldsymbol{\zeta}}_E^d + \underbrace{\boldsymbol{\lambda}_v^{E(i-1)}(\dot{\boldsymbol{\zeta}}_{i-1} - \dot{\boldsymbol{\zeta}}_i - \dot{\boldsymbol{\zeta}}_{E(i-1)}^d) + \boldsymbol{\lambda}_p^{E(i-1)}(\boldsymbol{\zeta}_{i-1} - \boldsymbol{\zeta}_i - \boldsymbol{\zeta}_{E(i-1)}^d)}_{\text{decentralizable}} \\ & + \mathbf{Q}_{EL}^{i}\boldsymbol{\omega}_L + \mathbf{H}_E^{i}\ddot{\boldsymbol{\zeta}}_E^d + \mathbf{Q}_E^{i}\dot{\boldsymbol{\zeta}}_E^d - \underbrace{\boldsymbol{\lambda}_v^{E(i)}(\dot{\boldsymbol{\zeta}}_i - \dot{\boldsymbol{\zeta}}_{i+1} - \dot{\boldsymbol{\zeta}}_{Ei}^d) - \boldsymbol{\lambda}_p^{E(i)}(\boldsymbol{\zeta}_i - \boldsymbol{\zeta}_{i+1} - \boldsymbol{\zeta}_{Ei}^d)}_{\text{decentralizable}}\end{aligned} \tag{47}$$

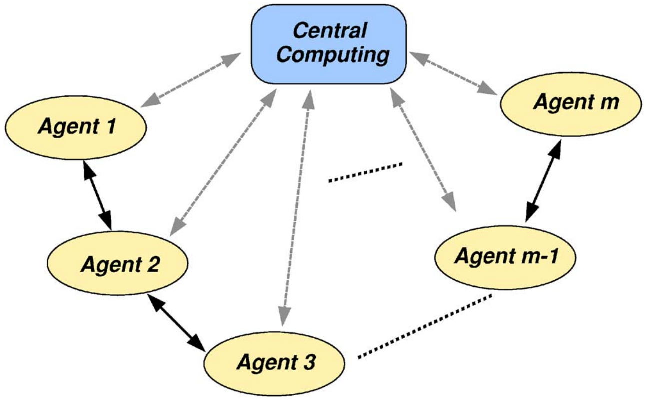

Fig. 3 Communication topology of centralized control: Decentralized control uses only decentralizable communication (black solid lines)

for $i=1,\ldots, m$, where arguments are omitted for brevity.

Some terms in (46) and (47) require state information of all agents, while the "decentralizable" terms require only those of its own and two neighboring agents (or one neighbor for the first and mth agents). In other words, implementation of (46) and (47) requires centralized communication, as shown in Fig. 3, where the black solid arrows represent information flow required for those "decentralizable" terms in (46) and (47), which can be computed/implemented by using just a local neighboring sensing or communication (i.e., distributed control implementation).

For the controller decentralization, we first decode the desired group formation and maneuver into desired trajectory of each agent. We assume that this desired trajectory is computed beforehand and stored in each agent. This desired trajectory is computed as follows: for the ith agent ($i=1,\ldots,m$), (1) desired translation trajectory $\mathbf{x}_i^d(t)\in\mathrm{Re}^3$ is (uniquely) obtained by solving

$$\frac{m_1\mathbf{x}_1^d+m_2\mathbf{x}_2^d+\cdots+m_m\mathbf{x}_m^d}{m_1+m_2+\cdots+m_m}=\mathbf{x}_L^d \quad \text{and } \mathbf{x}_j^d-\mathbf{x}_{(j+1)}^d=\mathbf{x}_{Ej}^d \tag{48}$$

for $j=1,\ldots, m-1$, where $\mathbf{x}_L^d(t)$ and $\mathbf{x}_E^d(t)=[\mathbf{x}_{dE(1)}^T,\ldots,\mathbf{x}_{dE(m-1)}^T]^T$ are the desired maneuver and formation of the group translation; and (2) similarly, desired attitude trajectory $\boldsymbol{\zeta}_i^d\in\mathrm{Re}^3$ for the ith agent ($i=1,\ldots,m$) is (uniquely) determined by

$$\sum_{i=1}^{m}\mathbf{A}_i\boldsymbol{\zeta}_i^d+\mathbf{b}=\boldsymbol{\zeta}_L^d \quad \text{and } \boldsymbol{\zeta}_j^d-\boldsymbol{\zeta}_{j+1}^d=\boldsymbol{\zeta}_{E(j)}^d \tag{49}$$

for $j=1,\ldots, m-1$, where $\boldsymbol{\zeta}_L^d\in\mathrm{Re}^3$ and $\boldsymbol{\zeta}_E^d=[\boldsymbol{\zeta}_{E(1)}^{dT},\ldots,\boldsymbol{\zeta}_{E(m-1)}^{dT}]^T$ are desired maneuver and formation of the group attitude, with $\mathbf{A}_i$, $\mathbf{b}$ being defined in (37).

With these precomputed desired trajectories $\mathbf{x}_i^d(t)$ and $\boldsymbol{\zeta}_i^d$ in (48) and (49) for each agent, we decentralize the translation and attitude controls (46) and (47) s.t. for $i=1,\ldots, m$,

$$\begin{aligned}\mathbf{t}_i &:= m_i\ddot{\mathbf{x}}_i^d(t)-\frac{m_i}{m_1+m_2+\cdots+m_m}[\mathbf{K}_v^L(\dot{\mathbf{x}}_i-\dot{\mathbf{x}}_i^d(t))+\mathbf{K}_p^L(\mathbf{x}_i-\mathbf{x}_i^d(t))]\\ &-\mathbf{M}_E^{i-1}\ddot{\mathbf{x}}_E^d(t)+\mathbf{K}_v^{E(i-1)}(\dot{\mathbf{x}}_{i-1}-\dot{\mathbf{x}}_i-\dot{\mathbf{x}}_{E(i-1)}^d(t))\\ &+\mathbf{K}_p^{E(i-1)}(\mathbf{x}_{i-1}-\mathbf{x}_i-\mathbf{x}_{E(i-1)}^d(t))+\mathbf{M}_E^i\ddot{\mathbf{x}}_E^d(t)\\ &-\mathbf{K}_v^{E(i)}(\dot{\mathbf{x}}_i-\dot{\mathbf{x}}_{i+1}-\dot{\mathbf{x}}_{E(i)}^d(t))-\mathbf{K}_p^{E(i)}(\mathbf{x}_i-\mathbf{x}_{i+1}-\mathbf{x}_{E(i)}^d(t))\end{aligned} \tag{50}$$

and

$$\begin{aligned}\boldsymbol{\tau}_i &:= -\boldsymbol{\lambda}_v^L\dot{\boldsymbol{\zeta}}_i-\boldsymbol{\lambda}_p^L(\boldsymbol{\zeta}_i-\boldsymbol{\zeta}_i^d)+\boldsymbol{\lambda}_v^{E(i-1)}(\dot{\boldsymbol{\zeta}}_{i-1}-\dot{\boldsymbol{\zeta}}_i)\\ &+\boldsymbol{\lambda}_p^{E(i-1)}(\boldsymbol{\zeta}_{i-1}-\boldsymbol{\zeta}_i-\boldsymbol{\zeta}_{E(i-1)}^d)-\boldsymbol{\lambda}_v^{E(i)}(\dot{\boldsymbol{\zeta}}_i-\dot{\boldsymbol{\zeta}}_{i+1})\\ &-\boldsymbol{\lambda}_p^{E(i)}(\boldsymbol{\zeta}_i-\boldsymbol{\zeta}_{i+1}-\boldsymbol{\zeta}_{E(i)}^d)\end{aligned} \tag{51}$$

where any terms having an index less than 1 or larger than m are to be zeros.

Here, we restrict the control objective for the decentralized attitude control (51) to be set-point regulation (i.e., both $\boldsymbol{\zeta}_L^d$ and $\boldsymbol{\zeta}_E^d$ are constant vectors). This is because trajectory tracking control requires information on the inertia and Coriolis terms of the decomposed dynamics (i.e., $\mathbf{H}_L(\boldsymbol{\zeta})$, $\mathbf{H}_E(\boldsymbol{\zeta})$,$\mathbf{Q}_L(\boldsymbol{\zeta},\boldsymbol{\omega})$, $\mathbf{Q}_E(\boldsymbol{\zeta},\boldsymbol{\omega})$, $\mathbf{Q}_{EL}(\boldsymbol{\zeta},\boldsymbol{\omega})$ and $\mathbf{Q}_{LE}(\boldsymbol{\zeta},\boldsymbol{\omega})$ in (47)), which are functions of the states of all agents, thus, generally not decentralizable. In contrast, such a restriction is not necessary for the decentralized translation control (50), since the translation dynamics (1) has only constant inertia matrices $\mathbf{M}_L$, $\mathbf{M}_E$ with no Coriolis terms. How to estimate such inertia matrices and Coriolis terms in a decentralized manner will be a topic for future work. The following theorem summarizes properties of the (stabilizing) decentralized controls (50) and (51).

THEOREM 2.

(1) *Consider the group translation dynamics* (1) *under the decentralized control* (50). *Suppose that the target accelerations* $\ddot{\mathbf{x}}_i^d(t)$ *is bounded and the disturbances* $\mathbf{f}_i(t)$ *are negligible (e.g., by local cancelation). Then,* $(\dot{\mathbf{x}}_L(t)-\dot{\mathbf{x}}_L^d(t),\mathbf{x}_L(t)-\mathbf{x}_L^d(t))\rightarrow\mathbf{0}$ *exponentially. Also,* $(\dot{\mathbf{x}}_E(t)-\dot{\mathbf{x}}_E^d(t),\mathbf{x}_E(t)-\mathbf{x}_E^d(t))$ *is ultimately bounded, whose bound can be made arbitrarily small by large formation gains* $\mathbf{K}_v^E$, $\mathbf{K}_p^E$ *in* (50);

(2) *Consider the group attitude dynamics* (2) *under the decentralized controls* (51). *Suppose that the inertia matrix* $\mathbf{H}_i(\boldsymbol{\zeta}_i)$ *and the Coriolis matrix* $\mathbf{Q}_i(\boldsymbol{\zeta}_i,\boldsymbol{\omega}_i)$ *in* (2) *are bounded. Suppose further that the disturbances* $\boldsymbol{\delta}_i$ *in* (2) *are negligible (e.g., by local cancelation), and target maneuver and formation* $\boldsymbol{\zeta}_L^d$ *and* $\boldsymbol{\zeta}_E^d$ *are all constant. Then,* $(\dot{\tilde{\boldsymbol{\zeta}}}_L(t),\tilde{\boldsymbol{\zeta}}_L(t)-\boldsymbol{\zeta}_L^d)\rightarrow\mathbf{0}$ *and* $(\dot{\boldsymbol{\zeta}}_E(t),\boldsymbol{\zeta}_E(t)-\boldsymbol{\zeta}_E^d)\rightarrow\mathbf{0}$ *exponentially.*

Proof.

(1) Let us define the following Lyapunov function $V_x(t)$ s.t.

$$\begin{aligned}V_x(t) &:= \sum_{i=1}^{m}\Bigg[\frac{1}{2}m_i(\dot{\mathbf{x}}_i-\dot{\mathbf{x}}_i^d)^T(\dot{\mathbf{x}}_i-\dot{\mathbf{x}}_i^d)+\frac{\phi_i}{2}(\mathbf{x}_i-\mathbf{x}_i^d)^T(\mathbf{K}_p^L\\ &+\epsilon\mathbf{K}_v^L)\\ &\times(\mathbf{x}_i-\mathbf{x}_i^d)+\epsilon m_i(\dot{\mathbf{x}}_i-\dot{\mathbf{x}}_i^d)^T(\mathbf{x}_i-\mathbf{x}_i^d)\Bigg]\\ &+\frac{1}{2}(\mathbf{x}_E-\mathbf{x}_E^d)^T(\mathbf{K}_p^E+\epsilon\mathbf{K}_v^E)(\mathbf{x}_E-\mathbf{x}_E^d)\end{aligned} \tag{52}$$

where $\phi_i:=(m_i)/(m_1+\cdots m_m)$ and $\epsilon>0$ is a small constant s.t. $V_x(t)$ is positive definite. Using the dynamics (1) with the decentralized control (50) and the condition (48), we then have

$$\frac{d}{dt}V_x(t) = -\sum_{i=1}^{m} \phi_i[(\dot{\mathbf{x}}_i - \dot{\mathbf{x}}_i^d)^T(\mathbf{K}_v^L - \epsilon\mathbf{M}_L)(\dot{\mathbf{x}}_i - \dot{\mathbf{x}}_i^d) + \epsilon(\mathbf{x}_i - \mathbf{x}_i^d)^T\mathbf{K}_p^L(\mathbf{x}_i - \mathbf{x}_i^d)] - (\dot{\mathbf{x}}_E - \dot{\mathbf{x}}_E^d)^T\mathbf{K}_v^E(\dot{\mathbf{x}}_E - \dot{\mathbf{x}}_E^d) - \epsilon(\mathbf{x}_E - \mathbf{x}_E^d)^T\mathbf{K}_p^E(\mathbf{x}_E - \mathbf{x}_E^d) + \sum_{i=1}^{m} \underbrace{[\mathbf{f}_i + (\mathbf{M}_E^i - \mathbf{M}_E^{i-1})\ddot{\mathbf{x}}_E^d]^T}_{\text{bounded}}[(\dot{\mathbf{x}}_i - \dot{\mathbf{x}}_i^d) + \epsilon(\mathbf{x}_i - \mathbf{x}_i^d)] \tag{53}$$

where we use the "telescoping" relation in the Appendix with $(\mathbf{a},\mathbf{b},\mathbf{G}) \in \{(\dot{\mathbf{x}},\dot{\mathbf{x}},\mathbf{K}_v^E),(\dot{\mathbf{x}},\dot{\mathbf{x}},\mathbf{K}_v^E),(\dot{\mathbf{x}},\dot{\mathbf{x}},\mathbf{K}_v^E),(\dot{\mathbf{x}},\dot{\mathbf{x}},\mathbf{K}_v^E)\}$, and the definition $\mathbf{M}_L = (m_1 + m_2 + \cdots m_m)\mathbf{I}_{3\times 3}$ from (30). Thus, by choosing small enough $\epsilon > 0$ s.t. $V_x(t)$ in (52) and $\mathbf{K}_v^L - \epsilon\mathbf{M}_L$ in (53) are all positive definite, $(\mathbf{x}_i - \mathbf{x}_i^d, \dot{\mathbf{x}}_i - \dot{\mathbf{x}}_i^d, \mathbf{x}_E - \mathbf{x}_E^d)$ is bounded $\forall t \geq 0$. Therefore, from (8), (29), and (48), $(\mathbf{x}_E - \mathbf{x}_E^d, \dot{\mathbf{x}}_E - \dot{\mathbf{x}}_E^d)$ and $(\mathbf{x}_L - \mathbf{x}_L^d, \dot{\mathbf{x}}_L - \dot{\mathbf{x}}_L^d)$ are bounded $\forall t \geq 0$.

Using the definitions of $\dot{\mathbf{x}}_L(t)$ and $\mathbf{x}_L(t)$ in (28) and (29) and the condition (48), we can show that the translation locked system dynamics (30) under the decentralized control (50) is given by:

$$\mathbf{M}_L(\ddot{\mathbf{x}}_L(t) - \ddot{\mathbf{x}}_L^d(t)) + \mathbf{K}_v^L(\dot{\mathbf{x}}_L(t) - \dot{\mathbf{x}}_L^d(t)) + \mathbf{K}_p^L(\mathbf{x}_L(t) - \mathbf{x}_L^d(t)) = \mathbf{f}_L \tag{54}$$

Therefore, if the disturbances $\mathbf{f}_i$ in (1) are negligible or properly canceled out, $(\mathbf{x}_L, \dot{\mathbf{x}}_L) \to (\mathbf{x}_L^d, \dot{\mathbf{x}}_L^d)$ exponentially.

We can also show that the shape system dynamics (31) with the decentralized control (50) is given by

$$\mathbf{M}_E(\ddot{\mathbf{x}}_E - \ddot{\mathbf{x}}_E^d) + \mathbf{K}_v^E(\dot{\mathbf{x}}_E(t) - \dot{\mathbf{x}}_E^d(t)) + \mathbf{K}_p^E(\mathbf{x}_E(t) - \mathbf{x}_E^d(t)) = \mathbf{f}_E + \sum_{i=1}^{m} g_i(\ddot{\mathbf{x}}_i^d(t), \dot{\mathbf{x}}_i(t) - \dot{\mathbf{x}}_i^d(t), \mathbf{x}_i(t) - \mathbf{x}_i^d(t)) \tag{55}$$

where the functions $g_i \in \mathrm{Re}^{3(m-1)}$, $i=1,\ldots,m$, are linear w.r.t. their respective arguments with no bias in the sense that $|g_i(t)| \leq a_1^i|\ddot{\mathbf{x}}_i^d(t)| + a_2^i|\dot{\mathbf{x}}_i(t) - \dot{\mathbf{x}}_i^d(t)| + a_3^i|\mathbf{x}_i(t) - \mathbf{x}_i^d(t)|$, $i=1,\ldots,m$, $\forall t \geq 0$ with a_1^i, a_2^i, $a_3^i > 0$ being all finite scalars. Thus, following (53) with the bounded $\ddot{\mathbf{x}}_i^d(t)$, $g_i(t)$ is bounded $\forall t \geq 0$. Therefore, with the negligible $\mathbf{f}_i$, the shape system dynamics (55) will be ultimately bounded and its bound can be made arbitrarily small by large enough gains $\mathbf{K}_p^E$, $\mathbf{K}_v^E$ in (50).

(2) Similar to (52), let us define the following Lyapunov function:

$$V_\zeta(t) := \sum_{i=1}^{m}\left[\frac{1}{2}\dot{\zeta}_i^T\mathbf{H}_i(\zeta_i)\dot{\zeta}_i + \frac{1}{2}(\zeta_i - \zeta_i^d)^T(\epsilon\boldsymbol{\lambda}_v^L + \boldsymbol{\lambda}_p^L)(\zeta_i - \zeta_i^d) + \epsilon\dot{\zeta}_i^T\mathbf{H}_i(\zeta_i - \zeta_i^d)\right] + \frac{1}{2}(\zeta_E - \zeta_E^d)^T(\boldsymbol{\lambda}_p^E + \epsilon\boldsymbol{\lambda}_v^E)(\zeta_E - \zeta_E^d)$$

where $\mathbf{H}_i(\boldsymbol{\zeta}_i)$ is the attitude inertia in (2) and $\epsilon > 0$ is a small constant so that $V_\zeta(t)$ is positive definite.

Differentiating $V_\zeta(t)$ w.r.t. time with the dynamics (2), the decentralized control (51), the condition (49), and the passivity property that $\dot{\mathbf{H}}_i - 2\mathbf{Q}_i = -(\dot{\mathbf{H}}_i - 2\mathbf{Q}_i)^T$ (or $\dot{\mathbf{H}}_i = \mathbf{Q}_i + \mathbf{Q}_i^T$), we can show that

$$\frac{d}{dt}V_\zeta(t) = -\underbrace{\sum_{i=1}^{m}[\dot{\zeta}_i^T(\boldsymbol{\lambda}_v^L - \epsilon\mathbf{H}_i)\dot{\zeta}_i + \epsilon\dot{\zeta}_i^T\mathbf{Q}_i(\zeta_i - \zeta_i^d) + \epsilon(\zeta_i - \zeta_i^d)\boldsymbol{\lambda}_p^L(\zeta_i - \zeta_i^d)]}_{=:V_\zeta^e(t)} - \dot{\zeta}_E^T\boldsymbol{\lambda}_v^E\dot{\zeta}_E - \epsilon(\zeta_E - \zeta_E^d)^T\boldsymbol{\lambda}_p^E(\zeta_E - \zeta_E^d) + \sum_{i=1}^{m}\boldsymbol{\delta}_i^T(\dot{\zeta}_i + \epsilon(\zeta_i - \zeta_i^d))$$

where, similar to (53), we use the "telescoping" relation in the Appendix having $(\mathbf{a},\mathbf{b},\mathbf{G}) \in \{(\dot{\boldsymbol{\zeta}},\dot{\boldsymbol{\zeta}},\boldsymbol{\lambda}_v^E),(\boldsymbol{\zeta},\boldsymbol{\zeta},\boldsymbol{\lambda}_p^E),(\boldsymbol{\zeta},\dot{\boldsymbol{\zeta}},\boldsymbol{\lambda}_v^E),(\boldsymbol{\zeta},\boldsymbol{\zeta},\boldsymbol{\lambda}_p^E)\}$ with the fact that $(d/dt)\boldsymbol{\zeta}_i^d = 0$ and $(d/dt)\boldsymbol{\zeta}_E^d = 0$. Thus, if the inertia matrices $\mathbf{H}_i$ and the Coriolis matrices $\mathbf{Q}_i$ are bounded, there always exists a small enough $\epsilon > 0$ so that $V_\zeta^e(t)$ in the above equality is positive definite. Therefore, using this small $\epsilon > 0$ with $\boldsymbol{\delta}_i = \mathbf{0}$, we have $(\dot{\boldsymbol{\zeta}}_i(t), \boldsymbol{\zeta}_i(t) - \boldsymbol{\zeta}_i^d) \to \mathbf{0}$ exponentially. Thus, from the condition (49), $(\dot{\tilde{\boldsymbol{\zeta}}}_L(t), \tilde{\boldsymbol{\zeta}}_L(t) - \boldsymbol{\zeta}_L^d) \to \mathbf{0}$ and $(\dot{\boldsymbol{\zeta}}_E(t), \boldsymbol{\zeta}_E(t) - \boldsymbol{\zeta}_E^d) \to \mathbf{0}$ exponentially. ■

With the decentralized controls (50) and (51), the communication requirement is now relaxed from the centralized communication to the neighboring communications (i.e., black solid arrows in Fig. 3). However, comparing Theorem 2 with Theorem 1, we can see that this controller decentralization has the following adverse effects/limitations: (1) For the group translation, the formation aspect would be perturbed by the desired maneuver acceleration (see (55)). In contrast, its maneuver is not affected by the decentralization at all (i.e., the locked system dynamics (54) is the same as that under the centralized control (30)); and (2) for the group attitude, we need to restrict the desired maneuver $\boldsymbol{\zeta}_L^d$ to be constant due to those "nondecentralizable terms" as explained in the paragraph before Theorem 2.

For the group attitude, since the decentralized control (51) does not cancel out the coupling terms $\mathbf{Q}_{EL}(\boldsymbol{\zeta},\boldsymbol{\omega})\boldsymbol{\omega}_L$ and $\mathbf{Q}_{LE}(\boldsymbol{\zeta},\boldsymbol{\omega})\dot{\boldsymbol{\zeta}}_E$ in (34) and (35), there would be crosstalk between the formation and maneuver, which are quadratic w.r.t. the operating speed. For the group translation, such crosstalk would affect only the formation (shape system) when the desired maneuver has nonzero acceleration (see (54) and (55)). Thus, both for the translation and attitude, if desired group behaviors are slow enough (i.e., small accelerations and velocities) so that such crosstalks become insignificant, precision of the formation and maneuver would still be satisfactory under the decentralized control.

As in the centralized control case (see the last paragraph of Sec. 4), due to the exponential convergence property of the decentralized control, the agent's state will be close to the desired one even under some perturbations (e.g., sensor and actuator noise, and model uncertainty), as long as their magnitude is small enough. Along the same reasoning in the paragraph after Theorem 1, for item 2 of Theorem 2, boundedness assumption on $\mathbf{H}_i(\boldsymbol{\zeta}_i)$ and $\mathbf{Q}_i(\boldsymbol{\zeta}_i,\boldsymbol{\omega}_i)$ will also be ensured, if $\boldsymbol{\omega}_i(0)$ is bounded.

6 Extension to an Equivalence Class of Formation Variables

In this section, we extend the results to the case where the formation variable (11) is given by an element in the following equivalence class of $\mathbf{q}_E$ in (11):

$$\mathcal{E}_{\mathbf{q}_E} := \{\mathbf{q}_E' \in \mathrm{Re}^{(m-1)n} \mid \exists\, \mathbf{E} \in \mathrm{Re}^{(m-1)n\times(m-1)n} \text{s.t.} \mathbf{q}_E' = \mathbf{E}\mathbf{q}_E\}$$

where $\mathbf{E}$ is a full-rank constant matrix. Here, to unify the translation and attitude cases, we use the notations of Sec. 3 with $\mathbf{q} \in \{\mathbf{x},\boldsymbol{\zeta}\}$. For example, consider $\mathbf{q}_E' := [\mathbf{q}_1^T - \mathbf{q}_2^T, \ldots, \mathbf{q}_1^T - \mathbf{q}_m^T]^T$ (i.e., agent 1 is the leader). This $\mathbf{q}_E' \in \mathcal{E}_{\mathbf{q}_E}$, since there is $\mathbf{E}$ s.t. its ith row is given by $[\mathbf{I},\ldots,\mathbf{I},\mathbf{0},\ldots,\mathbf{0}] \in \mathrm{Re}^{n\times(m-1)n}$ with i-copies of $\mathbf{I} \in \mathrm{Re}^{n\times n}$ and $(m-i-1)$-copies of $\mathbf{0} \in \mathrm{Re}^{n\times n}$. With the extension to this class of formation variables, as those in [9,37–39], our framework is not bound any more to a specific definition of the formation variables (e.g. (8) and (9)).

Now, suppose that, as the formation variable (11), we choose $\mathbf{q}_E' = \mathbf{E}\mathbf{q}_E \in \mathcal{E}_{\mathbf{q}_E}$. Similar to (12), we can then define the decomposition matrix $\mathbf{S}'(\mathbf{q})$ s.t.

$$\begin{pmatrix} \mathbf{v}_L' \\ \dot{\mathbf{q}}_E' \end{pmatrix} = \underbrace{\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{E} \end{bmatrix}}_{=:\mathbf{U}\in\mathrm{Re}^{mn\times mn}} \begin{pmatrix} \mathbf{v}_L \\ \dot{\mathbf{q}}_E \end{pmatrix} = \mathbf{U}\mathbf{S}(\mathbf{q})\dot{\mathbf{q}} = \mathbf{S}'(\mathbf{q})\dot{\mathbf{q}} \tag{56}$$

with $\mathbf{S}'(\mathbf{q}) := \mathbf{U}\mathbf{S}(\mathbf{q})$, where $\mathbf{S}(\mathbf{q}) \in \mathrm{Re}^{mn\times mn}$ and $\mathbf{v}_L \in \mathrm{Re}^n$ are the decomposition matrix and the locked system velocities with $\mathbf{q}_E$ in (11) as the formation variable, and those with $'$ are for $\mathbf{q}_E'$.

From the block-diagonal structure and being constant of $\mathbf{U}$ in (56), we can then decompose the group dynamics (10) similar to (22) and (23) s.t.

$$\mathbf{M}_L'(\mathbf{q})\dot{\mathbf{v}}_L' + \mathbf{C}_L'(\mathbf{q},\dot{\mathbf{q}})\mathbf{v}_L' + \mathbf{C}_{LE}'(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_E' = \mathbf{T}_L' + \mathbf{F}_L' \tag{57}$$

$$\mathbf{M}_E'(\mathbf{q})\ddot{\mathbf{q}}_E' + \mathbf{C}_E'(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_E' + \mathbf{C}_{EL}'(\mathbf{q},\dot{\mathbf{q}})\mathbf{v}_L' = \mathbf{T}_E' + \mathbf{F}_E' \tag{58}$$

where

$$\begin{bmatrix} \mathbf{M}_L' & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_E' \end{bmatrix} = \mathbf{U}^{-T}\begin{bmatrix} \mathbf{M}_L & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_E \end{bmatrix}\mathbf{U}^{-1}$$

$$\begin{bmatrix} \mathbf{C}_L' & \mathbf{C}_{LE}' \\ \mathbf{C}_{EL}' & \mathbf{C}_E' \end{bmatrix} = \mathbf{U}^{-T}\begin{bmatrix} \mathbf{C}_L & \mathbf{C}_{LE} \\ \mathbf{C}_{EL} & \mathbf{C}_E \end{bmatrix}\mathbf{U}^{-1} \tag{59}$$

and, similar to (17), using $(\mathbf{S}')^{-T} = \mathbf{U}^{-T}\mathbf{S}^{-T}$,

$$\begin{pmatrix} \mathbf{T}_L' \\ \mathbf{T}_E' \end{pmatrix} = (\mathbf{S}')^{-T}\mathbf{T} = \mathbf{U}^{-T}(\mathbf{S}^{-T}\mathbf{T}) = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{E}^{-T} \end{bmatrix}\begin{pmatrix} \mathbf{T}_L \\ \mathbf{T}_E \end{pmatrix} \tag{60}$$

with $\mathbf{T} = [\mathbf{T}_1^T,\ldots,\mathbf{T}_m^T]^T$. A similar relation also holds for $\mathbf{F}$. Here, note from (56)–(60) that the locked system is invariant w.r.t. the formation variable change (i.e., $\star_L' = \star_L$).

These decomposed systems (57) and (58) have the same structure and satisfy the Proposition 1 as their counterparts (22) and (23) do. Therefore, exactly same centralized control in Sec. 4 can be applied for (57) and (58). Suppose that, for (57) and (58), we design the following centralized control:

$$\mathbf{T}_L' = \mathbf{C}_{LE}'\dot{\mathbf{q}}_E' + \mathbf{M}_L'\ddot{\mathbf{q}}_L^d + \mathbf{C}_L'\dot{\mathbf{q}}_L^d - \mathbf{B}_L'(\mathbf{v}_L' - \dot{\mathbf{q}}_L^d) - \mathbf{K}_L'(\tilde{\mathbf{q}}_L - \mathbf{q}_L^d) \tag{61}$$

$$\mathbf{T}_E' = \mathbf{C}_{EL}'\mathbf{v}_L' + \mathbf{M}_E'\ddot{\mathbf{q}}_E^{d\prime} + \mathbf{C}_E'\dot{\mathbf{q}}_E^{d\prime} - \mathbf{B}_E'(\dot{\mathbf{q}}_E' - \dot{\mathbf{q}}_E^{d\prime}) - \mathbf{K}_E'(\mathbf{q}_E' - \mathbf{q}_E^{d\prime}) \tag{62}$$

where $\mathbf{q}_L^d$ and $\mathbf{q}_E^{d\prime} = \mathbf{E}\mathbf{q}_E^d$ are the desired trajectories and $\tilde{\mathbf{q}}_L'$ is the pseudo-configuration as in (37). Note that the controls presented in Sec. 4 are special examples of these (61) and (62). Here, we use $\mathbf{q}_L^d$ and $\tilde{\mathbf{q}}_L$ instead of $\mathbf{q}_L^{d\prime}$ and $\tilde{\mathbf{q}}_L'$, since $\mathbf{v}_L' = \mathbf{v}_L$ from (56).

Then, using (60), we can map $(\mathbf{T}_L',\mathbf{T}_E')$ to the controls $(\mathbf{T}_L,\mathbf{T}_E)$ for the original decomposed systems (22) and (23). First, the locked control (61) is mapped s.t.

$$\mathbf{T}_L = \mathbf{T}_L' = \mathbf{C}_{LE}\dot{\mathbf{q}}_E + \mathbf{M}_L\ddot{\mathbf{q}}_L^d + \mathbf{C}_L\dot{\mathbf{q}}_L^d - \mathbf{B}_L'(\mathbf{v}_L - \dot{\mathbf{q}}_L^d) - \mathbf{K}_L'(\tilde{\mathbf{q}}_L - \mathbf{q}_L^d)$$

This is because the locked system is invariant w.r.t. the formation variables as stated after (60) (i.e., $\star_L' = \star_L$) and $\mathbf{C}_{LE}'\dot{\mathbf{q}}_E' = \mathbf{C}_{LE}\mathbf{E}^{-1}\mathbf{E}\dot{\mathbf{q}}_E$ from (59). The shape control (62) is also mapped s.t.

$$\mathbf{T}_E = \mathbf{E}^T\mathbf{T}_E' = \mathbf{C}_{EL}\mathbf{v}_L + \mathbf{M}_E\ddot{\mathbf{q}}_E^d + \mathbf{C}_E\dot{\mathbf{q}}_E^d - \mathbf{B}_E(\dot{\mathbf{q}}_E - \dot{\mathbf{q}}_E^d) - \mathbf{K}_E(\mathbf{q}_E - \mathbf{q}_E^d)$$

where we use (59) with $\mathbf{C}_{EL}' = \mathbf{E}^{-T}\mathbf{C}_{EL}$, $\mathbf{v}_L' = \mathbf{v}_L$, $\mathbf{E}^{-1}\mathbf{q}_E' = \mathbf{q}_E$, and $\mathbf{E}^{-1}\mathbf{q}_E^{d\prime} = \mathbf{q}_E^d$. Here, we use the definitions $\mathbf{B}_E := \mathbf{E}^T\mathbf{B}_E'\mathbf{E}$ and $\mathbf{K}_E := \mathbf{E}^T\mathbf{K}_E'\mathbf{E}$, both of which are positive definite and symmetric, since $\mathbf{E}$ is full rank. This shows that a set of centralized controls designed such as (61) and (62) for *any* formation variable $\mathbf{q}_E' \in \mathcal{E}_{\mathbf{q}_E}$ are equivalent across different formation variables in $\mathcal{E}_{\mathbf{q}_E}$. In this sense, the centralized controls (61) and (62) constitute a class of equivalent controls.

Now, consider the decentralized controls (50) and (51) and denote them by the unified notation $\mathbf{T}_i$ ($\mathbf{T} \in \{\mathbf{t},\tau\}$). Using (17) and (18) as for (55), we can then show that these $\mathbf{T}_i$ can be mapped to $(\mathbf{T}_L,\mathbf{T}_E)$ of (22) and (23) s.t. $\mathbf{T}_L = \mathbf{M}_L\ddot{\mathbf{q}}_L^d - \mathbf{B}_L(\mathbf{v}_L - \dot{\mathbf{q}}_L^d) - \mathbf{K}_L(\tilde{\mathbf{q}}_L - \mathbf{q}_L^d)$ and $\mathbf{T}_E = \mathbf{M}_E\ddot{\mathbf{q}}_E^d - \mathbf{B}_E(\dot{\mathbf{q}}_E - \dot{\mathbf{q}}_E^d) - \mathbf{K}_E(\mathbf{q}_E - \mathbf{q}_E) + \mathbf{G}$, where some terms are zeros for the attitude control (51) and $\mathbf{G} \in \mathrm{Re}^{(m-1)n}$ is corresponding to the sum of (bounded) g_i functions in (55). Since they are in the same form as (61) and (62), the structure of these controls will be carried over to the control $(\mathbf{T}_L',\mathbf{T}_E')$ for (57) and (58) with $\mathbf{G}$ replaced by $\mathbf{G}' := \mathbf{E}^{-T}\mathbf{G}$ from (17). This implies that, under the decentralized controls (50) and (51), Theorem 2 also holds for other formation variables in $\mathcal{E}_{\mathbf{q}_E}$, too. Conversely, by converting it first to $(\mathbf{T}_L,\mathbf{T}_E)$, and then decentralizing it as in Sec. 5, we can also decentralize a centralized control $(\mathbf{T}_L',\mathbf{T}_E')$ for any formation variable $\mathbf{q}_E' \in \mathcal{E}_{\mathbf{q}_E}$ into the form (50) and (51) requiring only the line communication topology, as long as we choose the gains $\mathbf{B}_E'$ and $\mathbf{K}_E'$ for $\mathbf{T}_E'$ s.t. $\mathbf{B}_E := \mathbf{E}^T\mathbf{B}_E'\mathbf{E}$ and $\mathbf{K}_E := \mathbf{E}^T\mathbf{K}_E'\mathbf{E}$ are block diagonal as required in Sec. 5.

Perhaps, an even more interesting question would be how to decentralize the control according to the communication pattern implied by a formation variable $\mathbf{q}_E' \in \mathcal{E}_{\mathbf{q}_E}$, which may not necessarily be the line topology of Sec. 5. For instance, if $\mathbf{q}_E' = [\mathbf{q}_1^T - \mathbf{q}_2^T,\ldots,\mathbf{q}_1^T - \mathbf{q}_m^T]^T$, can we decentralize the control $(\mathbf{T}_L',\mathbf{T}_E')$ s.t. each decoded control $\mathbf{T}_i$ is a function of $\mathbf{q}_1$ and $\mathbf{q}_i$? Simple extensions of the proposed framework for this seem not to work at this moment. This is mainly because: (1) The decentralized controls (50) and (51) are derived w.r.t. the specific $\mathbf{q}_E$ in (11). Thus, simply by replacing the terms $\mathbf{q}_{E(i)}$ by $\mathbf{q}_{E(i)}'$ in (50) and (51), there is no guarantee that it also achieves the desired formation/maneuver and complies with the communication pattern implied by $\mathbf{q}_E'$; and (2) In Sec. 5, the matrix $\mathbf{S}^T(\mathbf{q})$ produces very naturally the decentralized controls (50) and (51) according to the communication pattern implied by $\mathbf{q}_E$. However, in general, this is not the case

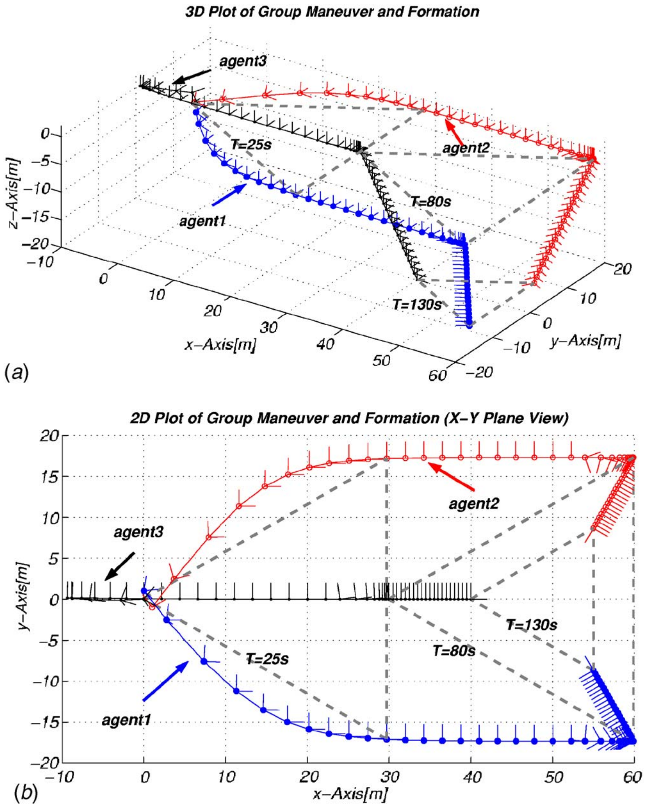

Fig. 4 3D and 2D simulation snapshots: Position and attitude of agent represented by sphere and body-fixed frame

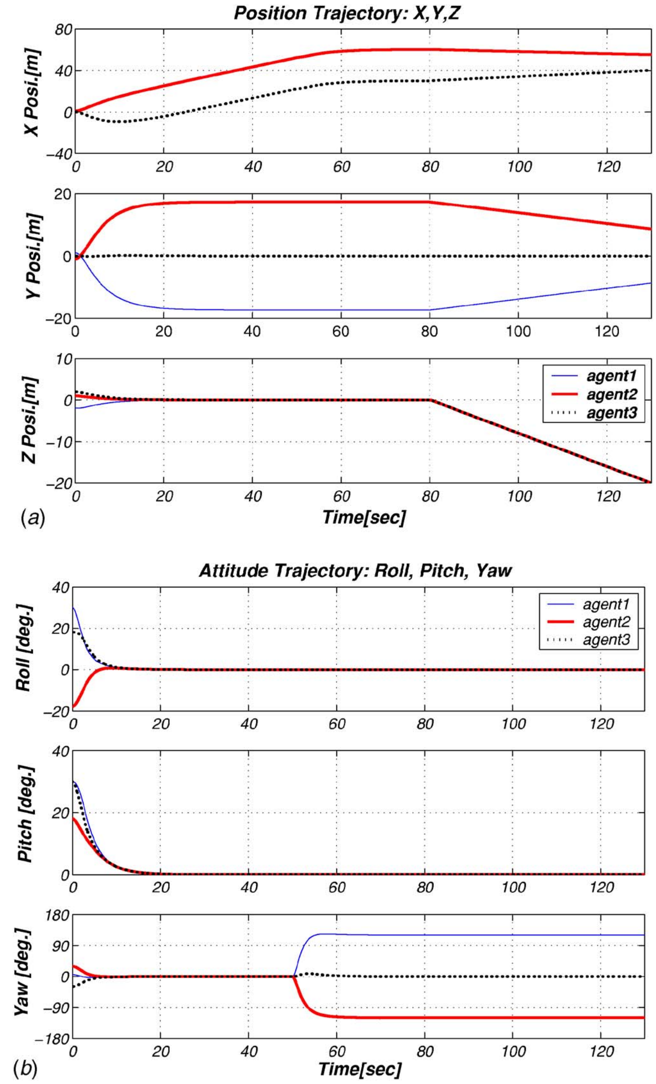

Fig. 5 Detailed data of the snapshots in Fig. 4

with other $(\mathbf{S}')^T$ and $\mathbf{q}'_E$, when $\mathbf{T}'_E$ contains the kinematic feedback terms (e.g., $\mathbf{K}'_E\mathbf{q}'_E$). How to achieve the controller decentralization for a given arbitrary communication structure is a topic for future research. For this, we will investigate a possible use of other tools, such as graph or matrix theory. See [40] for a result in this direction.

7 Simulation

7.1 Formation and Maneuver Control. We consider three agents whose dynamics is given by 6-DOF rigid body dynamics. We simulate each agent's dynamics using 6-DOF block of MatLab SimuLink Aerospace Blockset. Following [13], masses and principal-body-axis inertias of the three spacecraft are set to be: $m_1=20$ kg, $\mathbf{I}_1=\mathrm{diag}[0.73,0.55,0.63]$ k gm^2 for agent 1, $m_2=10$ kg, $\mathbf{I}_2=\mathrm{diag}[0.36,0.27,0.31]$ k gm^2 for agent 2, and $m_3=11$ kg, $\mathbf{I}_3=\mathrm{diag}[0.38,0.30,0.33]$ k gm^2 for agent 3. The centralized control in Sec. 4 is used and environmental disturbances are assumed to be negligible. Snapshots of the simulation are given in Fig. 4, where the position and attitude of each agent are represented by the sphere and the body-fixed (x,y,z)-frame. The body-fixed z-axis will be coincident with the inertial z-axis, when the pitch and roll angles are zeros. Simulation is performed for 130 s and snapshots are taken in every 5 s. Detailed simulation data are shown in Fig. 5 and its movie is available at http://www.me.umn.edu/~djlee/Formation/FF3DAng1.avi (or http://www.me.umn.edu/~djlee/Formation/FF3DAng2.avi for a different view angle).

We first derive the agent's dynamics w.r.t. the inertial frame. For the translation, each agent's dynamics is simply given by (1). For the attitude dynamics, since the inertia matrix is given w.r.t. the body-frame, we use the standard Jacobian map to get the dynamics w.r.t. the inertial frame in the form of (2). The group translation and attitude dynamics can then be decomposed as (30), (31), (34), and (35), respectively (see Sec. 3 for more details on the decomposition procedure). Since we assume negligible external disturbances, $\mathbf{f}_L$, $\mathbf{f}_E$, $\boldsymbol{\delta}_L$, and $\boldsymbol{\delta}_E$ in (30), (31), (34), and (35) are all zeros. By adjusting parameters in (32), (33), (39), and (40), we assign different controls $(\mathbf{t}_L,\mathbf{t}_E,\boldsymbol{\tau}_L,\boldsymbol{\tau}_E)$ on different time windows of the simulation, as briefly stated below. Once they are obtained, we decode them into individual agent's control $\mathbf{t}_i$ and $\boldsymbol{\tau}_i$ by using (17) for the translation and the attitude, separately.

During the first 50 s, to make an equilateral triangular formation shape (with $20\sqrt{3}m$ side-length), we choose $\mathbf{x}_E(t)=[0,-20\sqrt{3},0,30,10\sqrt{3},0]^T\in\mathrm{Re}^6$ with $\dot{\mathbf{x}}_E=0$. We also set $\mathbf{x}_L(t)=[v_x^d\times t,a_y,0]^T$ with $\mathbf{K}_p^L=\mathrm{diag}[0,k_p^L,k_p^L]$ $(k_p^L>0)$ so that the geometric center of the triangle floats along the x-axis with a constant speed v_x^d (=0.9 m/s) on a line $(y,z)=(0,0)$. Here, the (generally nonzero) constant offset a_y is necessary, since the center of the mass (described by the locked system) and the geometric center of the triangle (whose motion we want to control) are not the same. For the attitude controls (39) and (40), we choose $\tilde{\boldsymbol{\zeta}}_L=\boldsymbol{\zeta}_1$. We also set both $\boldsymbol{\zeta}_L^d(t)$ and $\boldsymbol{\zeta}_E^d(t)$ to be zero so that each agent's roll, pitch, and yaw angles are stabilized to zero (see Fig. 4 or 5).

From 50–80 s, the goal is to maintain the center of the triangle at $(50,0,0)$ m, while turning only the yaw angles of the agent 1 and agent 2 by $2\pi/3$ and $-2\pi/3$, respectively, so that all the

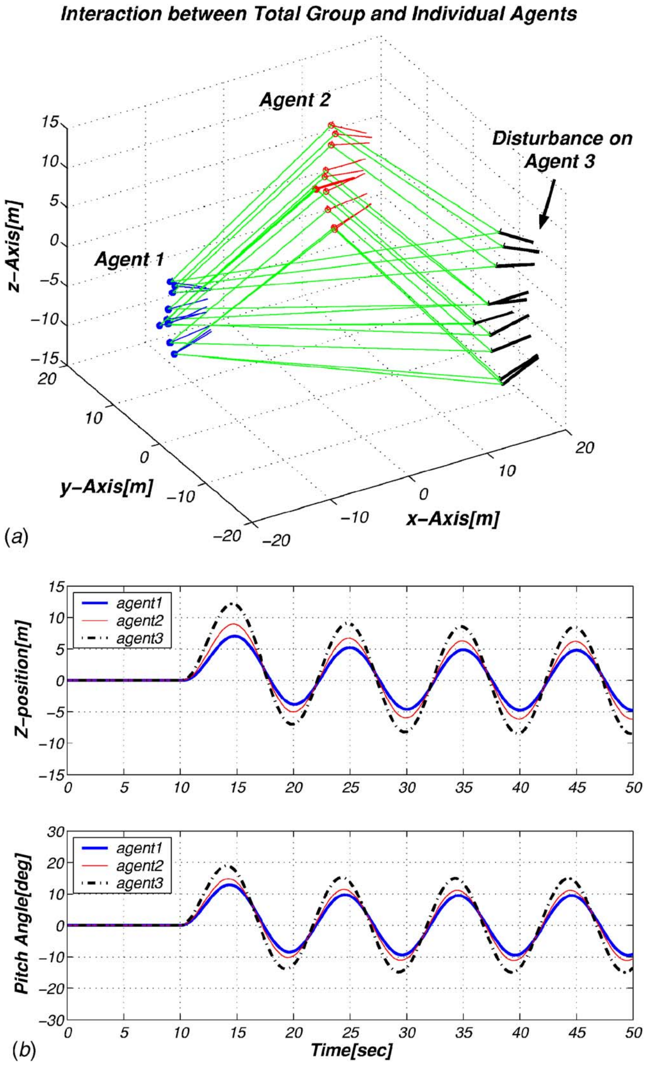

Fig. 6 Effect of disturbance on agent 3: Agent position/pitch angle represented by vertex of triangle and bar stemming from it

agents point to the center of the triangle $(50,0,0)$ [m]. Thus, from the previous control, we change the following: (1) $\mathbf{x}_L(t)=[50+a_x,a_y,0]^T$ with $\mathbf{K}_p^L=\text{diag}[k_p^L,k_p^L,k_p^L]$, where, similar to a_y, the constant offset a_x is used to compensate for the position difference between the geometric center of the triangle and the center of mass; and (2) $\boldsymbol{\zeta}_L^d(t)=[0,0,2\pi/3]^T$ and $\boldsymbol{\zeta}_E^d(t)=[0,0,4\pi/3,0,0,-2\pi/3]^T$. This choice of $\boldsymbol{\zeta}_L^d(t)$ is because we set $\tilde{\zeta}_L=\zeta_1$ (i.e., parametrize the locked system's attitude by that of the agent 1). See Sec. 4.2.

From 80–130 s, we aim to drive the equilateral triangle from $z=0$ m to $z=-20$ m while shrinking the triangle size and maintaining the agents' pointing to the triangle's center. Thus, we make the following change only on the previous translation control: $\mathbf{x}_L(t)=[50+a_x,a_y,-2ut]^T$ and $\mathbf{x}_E(t)=[0,-\sqrt{3}(20-ut),0,3(20-ut)/2,\sqrt{3}(20-ut)/2,0]^T$, where $u(=0.2\text{ m/s})$ is the shrinking rate.

We also performed the same simulation with the decentralized control in Sec. 5 and achieved similar results. We think that this is because the desired group behavior is slow enough so that the effect of the controller decentralization is not significant (see Sec. 5). We also injected several levels of sensor/actuator noises. However, as expected in Sec. 4 (or Sec. 5 for the decentralized con-

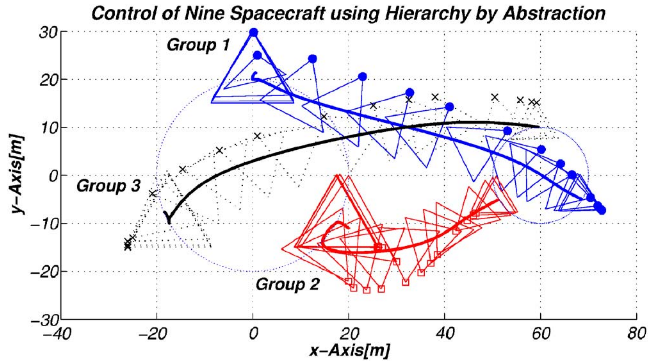

Fig. 7 Hierarchy by abstraction: Snapshots of nine agents, three groups (triangles), and their locked systems (solid lines)

trol), due to the exponential convergence property, the group behavior was still close to the desired one, as long as the noise level was small enough. Therefore, we omit here the simulation results with the decentralized control and the noise.

7.2 Group-Agents Interaction. The purpose of this simulation is to show that the group agents behave as one coherent group. We choose the same agents in Sec. 7.1 and use the decentralized control in Sec. 5. We impose sinusoidal disturbance on the agent 3 from 10 s, while the regulation controls stabilize the group translation and attitude to their respective set points. The dominant effects of the disturbance are in the directions of z-axis translation and pitch angle rotation. Data are plotted in Fig. 6. A movie of the simulation is also available at http://web.utk.edu/~djlee/movie/push.avi

As shown in Fig. 6, the perturbed state of the agent 3 affects the total group behavior and the states of other agents via the decentralized control. The disturbance propagates from the agent 3 to the agent 2 and the agent 1 through the control actions. Due to the damping actions in the controls, the disturbance effect is attenuated while propagating to other agents, as shown by the mitigated perturbed z-axis motion and pitch rotation of the agent 2 and the agent 1. Similar results were also achieved with the centralized control.

7.3 Hierarchy by Abstraction. For this simulation, we consider nine 2-DOF (x,y)-planar point masses. Utilizing the hierarchy with the centralized control (see Sec. 4.3 for more details), we partition them into three groups as shown in Fig. 7, where each triangle represents one group with three agents represented by its vertexes. We decompose each group by its locked and shape systems (L_i,S_i), $i=1,2,3$. The trajectory of each locked system L_i is shown by the solid line stemming from the center of each triangle.

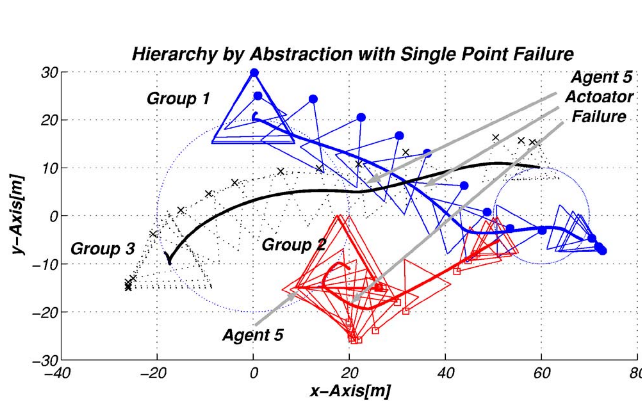

Fig. 8 Hierarchy by abstraction: Agent 5's actuation temporarily failed and recovered

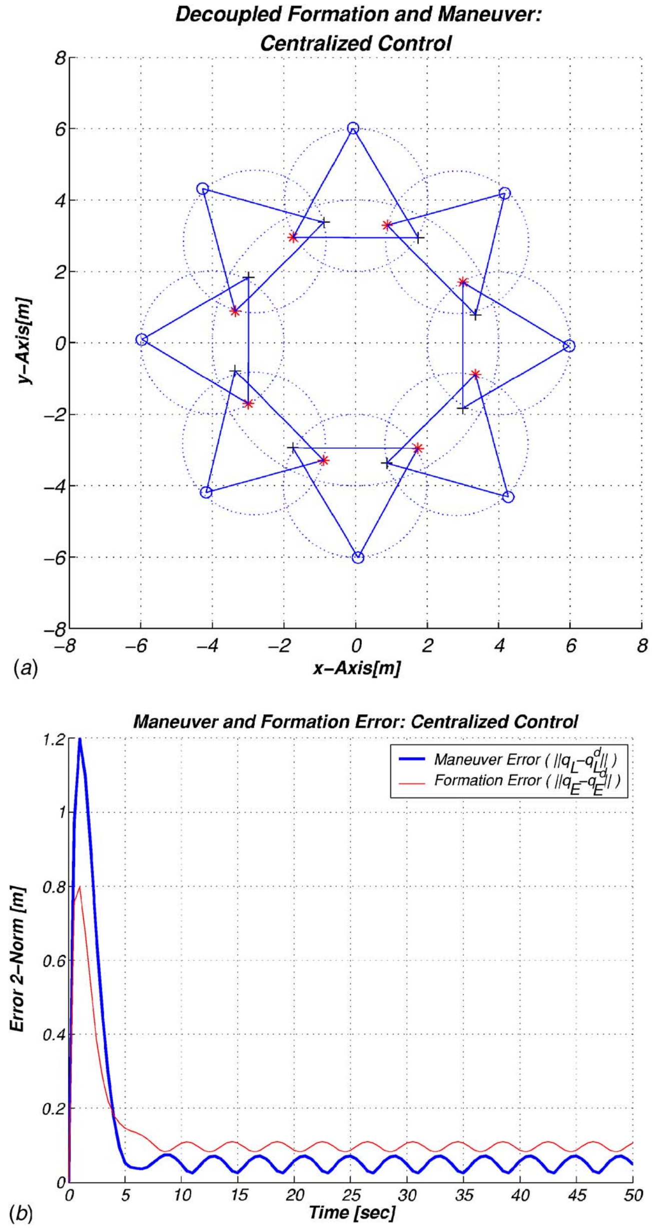

Fig. 9 Formation-maneuver decoupling with the centralized control

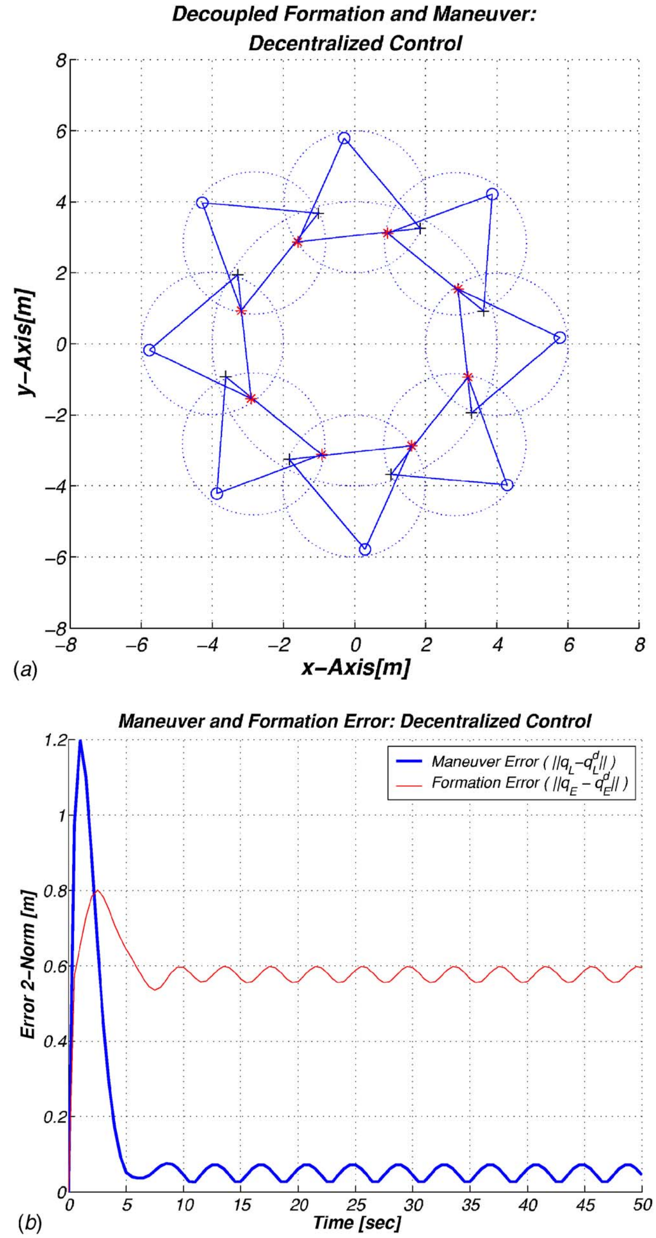

Fig. 10 Formation-maneuver coupling with the decentralized control

We also define the locked and shape systems $(\bar{L},\bar{S})$ of (L_1,L_2,L_3). The locked systems (L_1,L_2,L_3) then inherit their controls from those of $(\bar{L},\bar{S})$.

In Fig. 7, initially we stabilize $(\bar{L},\bar{S},S_1,S_2,S_3)$ to achieve three equilateral triangles with its center on the arc of the large circle (left). By controlling $\bar{L}$ and $\bar{S}$, we then drive the three triangles to the small circle (right), while shrinking the distance among the three equilateral triangles. At the same time, the shape systems (S_1,S_2,S_3) are controlled to shrink and rotate each equilateral triangle. Note from Fig. 7 that not only the group sizes are shrinking, but also the locations of the equilateral triangles are swapped on the circles. A movie of this simulation is also available at http://web.utk.edu/~djlee/movie/hier_nfail.avi

In Fig. 8, we use the same controls used in Fig. 7, but the actuation of agent 5 (group 2) has been temporarily deactivated. As shown in Fig. 8, to maintain the desired formation w.r.t. the agent 5, the remaining eight agents start to drift following the unactuated agent 5. However, as soon as the agent 5's actuation is activated again, all the nine agents are speeding up again to catch upto the desired trajectory, and eventually stabilized on the arc of the small circle. This simulation clearly shows that the multiple agents behave as one coherent group. A movie of this simulation is also available at http://web.utk.edu/~djlee/movie/hier_wfail.avi (or http://web.utk.edu/~djlee/movie/hier_wfail1.avi with deactivation during a different time period).

7.4 Effect of Decentralization. As shown in Sec. 5, under the decentralized control, the formation and maneuver are not decoupled from each other any more. This simulation is to show the effect of this incomplete decoupling of the decentralized controls.

We consider three point masses (1 kg) forming an equilateral triangle confined in the circle of 2 m radius. See Fig. 9 for an illustration. We design the formation and maneuver control so that the center of the triangle is revolving along the arc of the large circle with 4 m radius counter-clockwise with 8 s periodicity. The triangle itself is also rotating at the same rate so that one agent (marked with ∘) is always farthest from the center of the 4 m radius circle (i.e., on the circle of 6 m radius). We assume 10% mass identification error.

We first utilize the centralized control and Fig. 9 shows the simulation results. In the upper plot, the snapshots are taken with 5 s shooting speed during 10–50 s, while the lower plot shows the 2 norm of the maneuver and formation error. Similarly, the simulation results of the decentralized control are shown in Fig. 10.

For the translational dynamics, the desired maneuver acceleration perturbs the shape system (see item 1 of Theorem 2). This coupling effect induces the phase lag of the triangle rotation as shown by that the triangles are tilted in Fig. 10. However, with the centralized control in Fig. 9, this phase lag disappears, and the triangles maintain their rotation as their center rotates along the large circle.

Ripples in the lower plots of Figs. 9 and 10 are due to the mass identification error. With the perfect mass parameters, those ripples disappear, and, with the centralized control, the maneuver and formation errors converge to zero exponentially (or, with the decentralized control, the maneuver error converges to zero exponentially, while the formation error still maintains a constant bias). It is noteworthy to point out that the locked system dynamics and the maneuver error are not affected by the controller decentralization (see item 1 of Theorem 2). Such a preservation of the maneuver behavior would not be achieved for the attitude dynamics due to the couplings $\mathbf{Q}_{EL}$, $\mathbf{Q}_{LE}$ in (34) and (35).

8 Conclusion

In this paper, we propose a novel control framework for the formation of multiple rigid bodies. The key idea is the passive decomposition, with which we can decompose the nonlinear group dynamics into two decoupled systems: Shape system representing (internal group) formation aspect, and locked system abstracting (overall group) maneuver. By controlling them separately and individually, we can then achieve precise maneuver and formation controls simultaneously without any crosstalk between them. Furthermore, as the locked system is directly derived from the real dynamics of all agents, it would better abstract the group maneuver than an artificially-defined virtual system. We also connect group agents via bidirectional controls so that they can behave as one single coherent group (i.e., no runaway of slower agents). By abstracting a group by its locked system, we can also generate a hierarchy among multiple agents and groups. We also provide a controller decentralization which requires only undirected line communication graph topology, and show that performance of this decentralized control would still be satisfactory, if a desired group behavior is slow.

We believe that our centralized control scheme would be particularly promising for applications where high precision formation and maneuver are required (e.g., interferometry). We also believe that the decentralized control would be useful for applications where a desired group behavior is slow and only neighboring communication or sensing is available (e.g., agents with limited range on-board communication). We also think that the passive decomposition would give us a new way to analyze formation and maneuver for multiple agents with significant inertial effects. For a result in this direction, see [40].

Although we provide a controller decentralization, which requires only undirected line communication graph, we would like to investigate how to extend this result to an arbitrary digraph. Of our interest also is how to extend the proposed framework for the agents with nonholonomic constraints. We will also investigate applicability of the proposed framework for the orbital formation flying, where the passive decomposition is expected to achieve the formation and maneuver decoupling, while keeping the orbit and periodicity of the group motion. How to incorporate a collision avoidance capability into the proposed framework (while avoiding unwanted local minima) is also a topic for future research.

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Appendix

Consider $\mathbf{a}_i, \mathbf{b}_i, \mathbf{a}_i^d, \mathbf{b}_i^d \in \mathrm{Re}^n$, $i=1,\ldots,m$. Similar to (8), (9), (48), and (49), we can then define $\mathbf{a}_E, \mathbf{b}_E, \mathbf{a}_E^d, \mathbf{b}_E^d$ s.t. $\star_E := [\star_{E(1)}^T, \star_{E(2)}^T \ldots, \star_{E(m-1)}^T]^T \in \mathrm{Re}^{(m-1)n}$, where $\star \in \{\mathbf{a}, \mathbf{b}, \mathbf{a}^d, \mathbf{b}^d\}$ and $\star_{E(j)} = \star_j - \star_{j+1} \in \mathrm{Re}^n$, $j=1,\ldots,m-1$. Similar to the diagonal gain matrices in the controls (46) and (47), we also define a block-diagonal matrix $\mathbf{G} = \mathrm{diag}[\mathbf{G}^1, \mathbf{G}^2, \ldots, \mathbf{G}^{m-1}] \in \mathrm{Re}^{(m-1)n\times(m-1)n}$, where $\mathbf{G}^j \in \mathrm{Re}^{n\times n}$. The following "telescoping" relation then holds: With any term having an index less than 0 or larger than m being zero:

$$
\begin{aligned}
&\sum_{i=1}^{m} [(\mathbf{a}_i - \mathbf{a}_i^d)^T \mathbf{G}^{i-1} (\underbrace{\mathbf{b}_{i-1} - \mathbf{b}_i}_{=\mathbf{b}_{E(i-1)}} - \underbrace{(\mathbf{b}_{i-1}^d - \mathbf{b}_i^d)}_{=\mathbf{b}_{E(i-1)}^d}) - (\mathbf{a}_i - \mathbf{a}_i^d)^T \mathbf{G}^i (\underbrace{\mathbf{b}_i - \mathbf{b}_{i+1}}_{=\mathbf{b}_{E(i)}} - \underbrace{(\mathbf{b}_i^d - \mathbf{b}_{i+1}^d)}_{=\mathbf{b}_{E(i)}^d})] \\
&= -(\mathbf{a}_1 - \mathbf{a}_1^d)^T \mathbf{G}^1 (\mathbf{b}_{E(1)} - \mathbf{b}_{E(1)}^d) + (\mathbf{a}_2 - \mathbf{a}_2^d)^T \mathbf{G}^1 (\mathbf{b}_{E(1)} - \mathbf{b}_{E(1)}^d) - (\mathbf{a}_2 - \mathbf{a}_2^d)^T \mathbf{G}^2 (\mathbf{b}_{E(2)} - \mathbf{b}_{E(2)}^d) \\
&\qquad \vdots \\
&\quad + (\mathbf{a}_{m-1} - \mathbf{a}_{m-1}^d)^T \mathbf{G}^{m-2} (\mathbf{b}_{E(m-2)} - \mathbf{b}_{E(m-2)}^d) - (\mathbf{a}_{m-1} - \mathbf{a}_{m-1}^d)^T \mathbf{G}^{m-1} (\mathbf{b}_{E(m-1)} - \mathbf{b}_{E(m-1)}^d) - (\mathbf{a}_m - \mathbf{a}_m^d)^T \mathbf{G}^{m-1} (\mathbf{b}_{E(m-1)} - \mathbf{b}_{E(m-1)}^d) \\
&= \sum_{i=1}^{m-1} -(\mathbf{a}_i - \mathbf{a}_i^d - (\mathbf{a}_{i+1} - \mathbf{a}_{i+1}^d))^T \mathbf{G}^i (\mathbf{b}_{E(i)} - \mathbf{b}_{E(i)}^d) = -\sum_{i=1}^{m-1} (\mathbf{a}_{E(i)} - \mathbf{a}_{E(i)}^d)^T \mathbf{G}^i (\mathbf{b}_{E(i)} - \mathbf{b}_{E(i)}^d) = -(\mathbf{a}_E - \mathbf{a}_E^d)^T \mathbf{G} (\mathbf{b}_E - \mathbf{b}_E^d)
\end{aligned}
$$

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