Preliminary Results on Passive Velocity Field Control of Quadrotors

Changsu Ha, Francis B. Choi, and Dongjun Lee
Interactive & Networked Robotics Lab
School of Mechanical & Aerospace Engineering and SNU-IAMD
Seoul National University, Seoul, Korea
{changsuha,fbchoi,djlee}@snu.ac.kr

***Abstract*— We present preliminary results on the passive velocity field control (PVFC) of the quadrotor-type UAVs, which allows the quadrotor to follow the direction of a given velocity field while enforcing passivity of the closed-loop system. Some theoretical results are presented along with a simulation result.**

***Keywords*— backstepping, interaction, quadrotors, passive velocity field control, safety**

1. Introduction

In this paper, we present some preliminary results to achieve the behavior granted by the passive velocity field control (PVFC [1], [2]) on the quadrotor-type UAVs, which can guide the Cartesian motion of the quadrotor along the direction of a specified vector field, while enforcing the closed-loop quadrotor's passivity, thereby, enhances its interaction stability and safety with surrounding environments as well as possibly with human users. Backstepping control technique is used to address the issue of under-actuation of the quadrotors while extending the PVFC, which was originally developed for the fully-actuated robotic manipulators, for the quadrotors. Simulation is also performed to illustrate/verify the obtained theoretical results.

2. Modeling and Formulation

We consider a "mixed" quadrotor, consisting of Cartesian dynamics in E(3) and the attitude kinematics in SO(3):

$$m\ddot{x} = -\lambda R e_3 + m g e_3 + f_e \tag{1}$$

$$\dot{R} = R S(w_i) \tag{2}$$

where $e_k \in \Re^3$ are the basis vectors, with e_3 representing the down-direction, $m > 0$ is the mass, $x \in \Re^3$ is the quadrotor's position, $R \in SO(3)$ is the rotation matrix, $\lambda \in \Re$ is the thrust force input, $w := [w_1, w_2, w_3]^T \in \Re^3$ is the angular rate input, g is the gravitational constant, f_e is the external force including human/environment interaction force, and $S(\star) : \Re^3 \to so(3)$ is defined s.t. for $a, b \in \Re^3$, $S(a)b = a \times b$. This quadrotor is an under-actuated system, that is, its dynamics is 6-dimensional, yet, with only the 4 controls (i.e., λ, w_1, w_2, w_3), which makes the control design involved. See [3], [4] for more details on the quadrotor dynamics.

Research supported in part by the Basic Science Research Program (2012-R1A2A2A0-1015797).

We also design the velocity field, that is, a (constant) vector $V(x)$ assigned on each Cartesian position of the quadrotor $x \in \Re^3$, s.t.,

$$V : \mathscr{X} \mapsto T_x\mathscr{X}; \quad x \in \mathscr{X} \mapsto V(x) \in T_x\mathscr{X}$$

where $\mathscr{X} \approx \Re^3$ is the configuration manifold of the quadrotor's E(3)-dynamics (1), and $T_x\mathscr{X} \approx \Re^3$ is the tangent space at $x \in \mathscr{X}$. This V then defines a desired *direction* of the quadrotor's velocity $\dot{x}$ at each $x \in \Re^3$. We also design V to be of a constant energy $E > 0$ s.t.,

$$E = \tfrac{1}{2} m V(x)^T V(x), \quad \forall x \in \Re^3.$$

What we want is then to achieve $\dot{x}$ to follow V s.t., when $f_e = 0$,

$$e_\beta := \dot{x} - \beta V \to 0 \tag{3}$$

where $\beta \geq 0$ defined s.t.

$$\beta^2 := \frac{K}{E} = \frac{\frac{1}{2}\dot{x}^T m \dot{x}}{\frac{1}{2} V^T m V}$$

and $K := \frac{1}{2} m \dot{x}^T \dot{x}$. Note that, with this β, (3) implies that $\dot{x}$ will follow V (i.e., convergence), with its speed determined by K (i.e., passivity). The PVFC controller, to be presented in Sec. 3, will enforce this (passive) behavior of the closed-loop quadrotor, that is: 1) it will follow $V(x)$ only if it contains enough energy K in it; and 2) if the system loses all of its energy (e.g., dragging obstacles (or humans) during the operation), the quadrotor will eventually stop. The latter behavior particularly means that the PVFC-controlled quadrotor would be safer to interact with, since the (energetic) damage induced by the quadrotor on the obstacles/humans will be limited. Refer to [1], [2], [5] for more details on this safe interaction of PVFC.

3. Main Results

In this section, we present the main theoretical results and some illustrative simulation results. Due to the page limit, we omit many details on them, which we will report in future publications.

3.1 *The Control and its Theoretical Properties*

While extending the PVFC [1], [2] to the quadrotor, we need to address the under-actuation of the quadrotor (1)-(2), for which, similar to [3], [6], we utilize the backstepping

technique. We can then design the following backstepping-based PVFC control law for the quadrotor:

$$[\lambda w_2, -\lambda w_1, \dot{\lambda} + \alpha\lambda]^T \tag{4}$$
$$= -\left[\frac{d}{dt} + \alpha\right] \cdot (\tau_c + \tau_f) + \alpha m g e_3 + \rho \left[e_\beta - \frac{m\dot{x}}{2\beta E} e_\beta^T V\right]$$

where $\frac{d}{dt}$ is the differentiation operator, $\alpha, \rho > 0$ are gains,

$$\tau_c(x,\dot{x}) := \frac{1}{2E}(\omega \mathbb{P}^T - \mathbb{P}\omega^T)\dot{x} =: \mathbb{G}(c,\dot{x})\dot{x}$$
$$\tau_f(x,\dot{x}) := \gamma(\mathbb{P}p^T - p\mathbb{P}^T)\dot{x} =: \gamma\mathbb{R}(x,\dot{x})\dot{x}$$

where

$$\mathbb{P} := mV, \quad w := m\dot{V} = m(\partial V/\partial x)\dot{x} = \dot{\mathbb{P}}, \quad p := m\dot{x}$$

and $\gamma > 0$ is the feedback gain. Here, note that $\mathbb{G}, \mathbb{R} \in \Re^{3\times 3}$ are skew-symmetric, implying that, when coupled with $\dot{x}$, τ_c, τ_f do not produce/dissipate any energy.

Theorem 1: Consider the quadrotor (1)-(2) with the backstepping PVFC control (4). Then, given $(e_\beta(0), v_e(0))$, there always exists $\underline{v}(e_\beta(0), v_e(0)) > 0$ s.t., if $||\dot{x}(t)|| > \underline{v}$ $\forall t \geq 0$, $(e_\beta(t), v_e(t)) \to 0$ exponentially if $f_e = 0$; or $(e_\beta(t), v_e(t))$ is ultimately-bounded if f_e is bounded, whose bounded can be made arbitrarily small by increasing γ, α.

This Th. 1 states that, the smaller the initial error $(e_\beta(0), v_e(0))$ is, the smaller the quadrotor speed $||\dot{x}(t)||$ can be while still enforcing the exponential stability of $(e_\beta, v_e) = 0$. The following Prop. 1 show that, with some physical viscous damping, the PVFC-controlled quadrotor can be guaranteed to be (energetically) passive, thus, safer to interact with.

Proposition 1: Consider the quadrotor (1)-(2) under the backstepping-based PVFC control (4). Assume $f_e = f_h - b\dot{x}$, where f_h is the interaction force and b is the quadrotor's (minimum) physical viscous damping. Then, if we choose b/ρ large enough s.t. $b\alpha/\rho \geq (3/2)^2$, the closed-loop quadrotor is (energetically) passive through its Cartesian interaction port $(f_h, \dot{x})$.

3.2 *Simulation Results*

We perform a simulation with the mixed quadrotor (1)-(2) under the backstepping-based PVFC control (4). We also include artificial damping b to emulate the physical aerodynamic viscous damping. The results are presented in Fig. 1, where the top plot shows the planar velocity field in 3D and the quadrotor trajectory; and the bottom plot shows the velocity field following error $||e_\beta|| = ||\dot{x} - \beta V||$, the quadrotor's kinetic energy K, and the deviation distance from the 1[m]-radius circle $||e_{\text{traj}}||$, which defines the limit-cycle behavior of the vector field.

From Fig. 1, we can then see that: 1) the quadrotor follows the vector field $V(x)$ whenever it has enough kinetic energy K; 2) it yet slows down and stops (while still on the unit circle) due to the damping dissipation via b with kinetic energy K also depleting (around 10s and 20s); 3) it speeds up again and resumes the velocity following with the velocity determined by K, when the energy is injected by the impulse

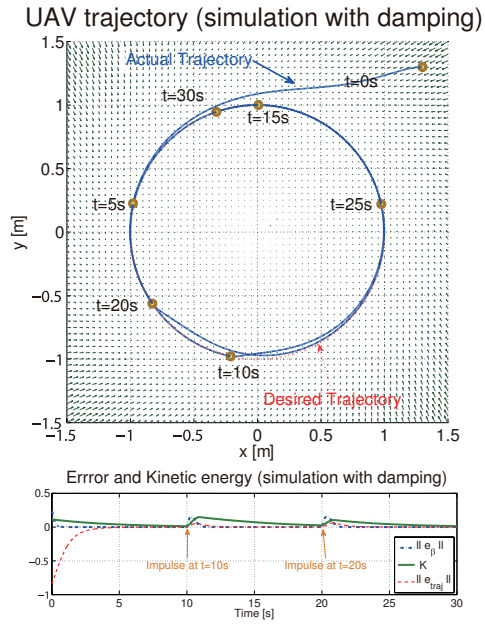

Fig. 1. Simulation results with artificial damping injection and occasional human pushes.

applied at 10s and 20s; and 4) all the error measures, $||e_\beta||$ and $||e_{\text{traj}}||$, exponentially converge to zero whenever the quadrotor has enough kinetic energy.

4. Conclusion and Future Research Topics

The development, presented in this paper, is only preliminary, and we are currently working to improve/extend it for realistic and practical application scenarios, particularly: 1) to overcome the effect of uncertainty (e.g., vertical drift) by using feedback or adaptive action; 2) to optimize controller performance via, e.g., scaling the vector field $V(x)$; 3) to include fictitious energy storage to directly control/modulate the energy available in the system; 4) to fully analyze interaction through the attitude dynamics; and 5) implementation and experimentation particularly with human-interaction.

References

[1] P. Y. Li and R. Horowitz. Passive velocity field control of mechanical manipulators. *IEEE Transactions on Robotics and Automation*, 15(4):751–763, 1999.

[2] P. Y. Li and R. Horowitz. Passive velocity field control: Part 1. geometry and robustness. *IEEE Transactions on Automatic Control*, 46(9):1346–1359, 2001.

[3] D. J. Lee, C. Ha, and Z. Zuo. Backstepping control of quadrotor-type uavs: trajectory tracking and teleoperation over the internet. In *Proc. Int'l Conf. on Autonomous Systems*, 2012.

[4] M-D. Hua, T. Hamel, P. Morin, and C. Samson. A control approach for thrust-propelled underactuated vehicles and its application to vtol drones. *IEEE Transactions on Automatic Control*, 54(8):1837–1853, 2009.

[5] D. J. Lee and P. Y. Li. Passive bilateral control and tool dynamics rendering for nonlinear mechanical teleoperators. *IEEE Transactions on Robotics*, 21(5):936–951, 2005.

[6] D. J. Lee. Distributed backstepping control of multiple thrust-propelled vehicles on balanced graph. *Automatica*, 2012. To appear.