Passivity-Based Control of Manipulator-Stage Systems on Vertical Flexible Beam

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Abstract—We develop a novel passivity-based control framework of a manipulator-stage system sitting on a vertical flexible beam. We first model the vertical flexible beam by using Euler-Bernoulli theory and modal approximation while utilizing certain boundary conditions to take into account the effect of the gravity and interaction wrench between the manipulator-stage system and the beam. To separately achieve two control objectives, i.e. the joint tracking and the vibration suppression, we utilize the passive decomposition to split the dynamics into: 1) the stage and beam dynamics; and 2) its orthogonal complement, which converges to the robotic manipulator dynamics as the vibration is subdued. We further show that the linearized stage-beam dynamics is controllable and design an LQR control to stabilize the beam vibration only by using the stage motion. We also design control for the orthogonal complement dynamics in such a way that it can attain the manipulator joint tracking asymptotically as the vibration is suppressed. Experiments are also performed to verify the manipulator joint tracking while the vibration is suppressed internally via the stage motion.

I. INTRODUCTION

According to the International Atomic Energy Agency (IAEA), there are more than 400 plants operating in 30 countries. In particular, CANDU (CANada Deuterium Uranium) reactor, a heavy-water nuclear power plant, is designed with a unique function: two machines feed the reactor during operation. See (b) in Fig. 1. However, if fuel is caught in the fuel supply of the machine, emergency repairs must be performed. The problem is that the skilled worker has to wait until the radiation dose decrease and then put the unique device attached to the end of the long rod tool onto the emergency shaft at a height of 10-15 meters. After coupling between the device and the shaft, the operator must rotate the shaft with sufficient force. The ultimate goal of this research is to develop a remote-acting robot system so we can use it for any emergency repairs in nuclear power plants that are time-consuming and dangerous to human technicians.

For performing emergency tasks, the robotic system should have sufficient torque capacity and be able to reach a 10 m height. Also, to put the special tool to the emergency shaft, the precise motion is significant. Even though aerial manipulator systems which have recently received much attention would be one option [1], [2], they are not suitable because of their limited payload and flight time to accomplish these emergency tasks. Thus, we are developing the manipulator-stage system sitting on a vertical beam which has long height as depicted in (a) of Fig. 1. We choose this system as a combination of a dual robotic arm and a linear stage. Each is used for different purposes as explained later. The challenge is that the flexibility of the vertical support cannot be ignored because of its length, implies that the manipulator motion or external forces can cause the vibrational motion of the support.

In this paper, we aim to design a novel passivity-based control framework for such manipulator-stage system sitting on the vertical flexible beam. The control goal is to attain the desired operation of the manipulator while suppressing the vibration. To achieve the objective, we first model the vertical beam by using Euler-Bernoulli theory with certain boundary condition. Note that, before deploying the system in (a) of Fig 1, we consider simplified version as shown in Fig 2 so that we can first derive the core theoretical methodologies and check their feasibility. The simplified version consists of planar three degree of freedom (DOF) manipulator with one DOF linear stage and aluminum vertical beam. We then derive the dynamics of the entire system consisting of the flexible beam, the linear stage and the robotic arm. Utilizing the passive decomposition [3] facilitates to split the dynamics into the stage-beam dynamics and its orthogonal complement, that converges to the manipulator dynamics as the vibration is suppressed. The dynamically decomposed dynamics allow the separate design of the tracking control
and the vibration suppression. Our main idea of control design is to use a linear stage to reduce vibration as quickly as possible and let the manipulator follow the desired trajectory through a passivity-based control. For stage input, we analyze the controllability of the linearized stage-beam dynamics of which control input is only the stage motion and then design the vibration suppression control for the stage. Also, we design the passivity-based joint trajectory tracking control in the orthogonal complement dynamics.

Our contributions are 1) to utilize the passive decomposition for an underactuated and flexible system, 2) to suggest a novel control framework for manipulator systems on compliant base, which facilitates separate control design of two different control objectives, i.e. joint tracking and vibration suppression, and 3) to perform the experiment to show the performance of the vibration suppression and the tracking even under the external excitation. As for the passive decomposition, authors in [4], [5] use the passive decomposition for the fully-actuated and rigid-body systems in, yet, none of them has been demonstrated for underactuated and flexible systems. Many studies have dealt with the control problem of manipulators on flexible supports from mid of 90’s. Macro/micro-manipulators can be considered as one of these types of problems due to the flexibility of the macro manipulator [6–8]. Another researches consider the compliant base problem in [9–11]. However, most of the above use the mixed control method that puts the two different control inputs into one control channel by assuming that the time scale of vibration and tracking is different. To avoid this one-channel control approach, redundant systems are utilize in [12]. In [12], the vibration suppression is first designed, and null-space motion is used for tracking control. However, this control is also capable of either tracking or vibration suppression. Also, whereas we model the flexibility as rigorous as possible, in most cases the flexible base is simplified to model as a single DOF, such as a spring and damper system.

The rest of the paper is organized as follows. Sec. II shows the modeling of the manipulator-stage system on the vertical flexible beam. Sec. III presents the dynamics decomposition of the total system, control design, and controllability analysis of the linearized stage-beam dynamics. We demonstrate the performance of desired joint trajectory tracking and vibration suppression control in Sec. IV. Finally, we summarize the proposed results with some comments on the future research directions in Sec. V.

II. SYSTEM MODELING

To model the flexible beam, we utilize Euler-Bernoulli theory with a modal approximation. Then, we rigorously consider the boundary condition of our system configuration to calculate the natural frequency of the assumed finite modes. The mode shapes are considered as eigenfunctions of the flexible beam so that enforcing the orthogonality condition between the mode shapes is possible to specifically determine the mode shapes. Finally, we use Euler-Lagrange equations to derive the dynamics model of the whole system with the calculated mode shapes.

A. System Description

Let us define following six frames (See. Fig. 2): the ground fixed frame \( \{O\} \) which is located in the bottom of the beam, the flexible moving frame \( \{B\} \) which is attached at the end of the beam, the body frame of the linear stage \( \{S\} \), and the body frame of each joint \( \{1\}, \{2\}, \) and \( \{3\} \). Among any two of these frames, we can then define \( p_{\text{obs}} \) is the position vector of the origin of \( \{B\} \) from the origin of \( \{A\} \) expressed in \( \{A\} \).

We define \( l_b \) as the length of the beam. The rotational angle and lateral deflection at the tip of the beam is expressed by \( \theta_b \) and \( w(l_b,t) \), respectively. If there is no ambiguity, we will use \( w \) for the deflection at the tip. The x-axis is the upward direction, which defines the direction of the beam as the x-axis. We assume that the deflection of the beam is small enough that the following approximations are valid.

\[
p_{\text{obs}} \approx \begin{bmatrix} l_b \\ w(l_b,t) \end{bmatrix} \in \mathbb{R}^2, \quad \theta_b \approx \frac{dw}{dx}(l_b,t) \in \mathbb{R}
\]

We will use subscripts \( \{s,1,l_1,2,l_2,3,l_3\} \) which indicate stage, the motor 1, link 1, ..., link 3. For instance, \( m_{l_3} \) means the mass of the link 2. Each motion of the actuator is denoted by \( q_i \) where \( i \in \{s,1,2,3\} \).  

B. Assumed Mode Shapes

Based on Euler-Bernoulli theory, govern equation of the lateral vibration motion of beams and general solutions [13] are given by

\[
\rho A \frac{d^2 w}{dt^2}(x,t) + \frac{d^2}{dx^2} \left( EI(x) \frac{d^2 w}{dx^2}(x,t) \right) = 0 \quad (1)
\]

\[
w(x,t) = \sum_{i=1}^{\infty} \phi_i(x) \delta_i(t) \approx \sum_{i=1}^{n_d} \phi_i(x) \delta_i(t) \quad (2)
\]

where \( \rho \) is density, \( A \) is intersection area, \( E \) is Young’s modulus, \( I(x) \) is second moment of inertia of the beam,
and \( n_d \) is the number of the assumed modes. This paper uses \( n_d = 3 \) because of its high payload. In other words, the high-frequency mode is virtually invisible due to the mass and inertia of the stage manipulation system. Furthermore, for simplicity, we assume that all parameters of the beam are constant. The eq. (2) presents separated variable solution where \( \phi_i(x) \) is \( i \)-th time-invariant mode shape and \( \delta_i(t) \) is time-varying \( i \)-th mode’s amplitude.

It is well-known the explicit form of \( i \)-th mode shape is

\[
\phi_i(x) = C_1 \sin \beta_i x + C_2 \cos \beta_i x + C_3 \sinh \beta_i x + C_4 \cosh \beta_i x \tag{3}
\]

where \( \beta_i^2 = \omega_i^2 \rho A/EI \), \( \omega_i \) is natural frequency of \( i \)-th mode, and each \( C_i \) is a coefficient. Note that the mode shapes are time-invariant and determined by boundary conditions, which are 1) clamped ground-end and 2) lumped mass/inertia tip-end conditions as depicted in Fig. 3.

1) Boundary Conditions: The two boundary conditions can be expressed by the following four equations

\[
w(0, t) = 0, \quad \frac{dw}{dx}(0, t) = 0 \tag{4}
\]

\[
M(b_l) = EI \frac{d^2 w}{dx^2}(b_l, t) = -J_0 \theta_0 - M_d \ddot{w} - M_g \ddot{g} \tag{5}
\]

\[
V(b_l) = -EI \frac{d^2 w}{dx^2}(b_l, t) - m_0 \ddot{w} - M_d \ddot{g} \tag{6}
\]

where \( M(b_l) \) is the moment and \( V(b_l) \) is the shear force applied to the tip of the beam, respectively. In addition, \( J_0 = \sum_{i=s}^{l_0} J_i \) and \( m_0 = \sum_{i=s}^{l_0} m_i \) are sum of total moment of inertia and mass of the rigid part, respectively. \( g = 9.81 \text{m/s}^2 \) is gravitational acceleration, \( M_d = \sum_{i=s}^{l_0} m_i |p_i| \), and \( M_g = \sum_{i=s}^{l_0} m_i |p_0b_2| \). Here, \([s] \) is the \( y \)-axis component of the vector \( * \in \mathbb{R}^2 \). The boundary conditions (5) and (6) presents that the tip of the beam receives the moment and shear force because of the motion of the robotic parts and gravity.

2) Calculation of Natural Frequencies: One can obtain \( C_1 = -C_3 \) and \( C_2 = -C_4 \) from the first boundary condition eq. (4). Then, the second boundary conditions eq. (5) and (6) lead to the following homogeneous equation

\[
[F(\beta_i)] [C_3] = 0
\]

where \([F(\beta_i)]\) is 2 by 2 matrix whose elements are transcendental functions of \( \beta_i \). We can obtain \( \beta_i \) by using

\[
\det(F) = 0, \quad \text{which is usually called frequency equation, to exclude trivial solution.}
\]

Note that the second boundary condition is actually non-homogeneous due to the term \( M_g \dot{g} \), which implies that we cannot use the frequency equation to obtain \( \beta_i \). However, we assume that the gravity effect on the natural frequency is negligible. Moreover, \( M_d \) in (5) and (6) is a function of the configuration of the rigid part so that we should calculate the natural frequencies whenever configurations of the manipulator-stage system are changed. However, as stated in [14], we find that the variation of the natural frequency corresponding to the configuration change of the manipulator-stage system is little. Thus, we take the natural frequency when the manipulator is at the zero-configuration, i.e., all joint angles are zero.

3) Normalization and Orthogonality of Mode Shapes: The mode shapes are eigenfunctions which describe the motion of the beam, thus we should enforce the orthogonality condition between the mode shapes [13]. The orthogonality leads to the diagonal effective stiffness in the system dynamics, which will be derived in Sec. II-C. Furthermore, four eqs. (4) to (6) of the boundary conditions are insufficient to determine five unknowns, \( \beta_i \) and four \( C_i \), in (3). To specify these unknowns, normal modes are used [13].

\[ T = T_b + T_s + \sum_{i=1}^{l_s} T_i \]

where \( T_b, T_s, \) and \( T_i \) are the kinetic energy of the beam, stage, and the motor/link, respectively. Each kinetic energy is specifically defined by

\[ T_b = \frac{\rho A}{2} \int_0^{l_b} \dot{p}_{ob}(x)^T \dot{p}_{ob}(x) dx = \frac{\rho A}{2} \int_0^{l_b} \dot{w}(x, t)^2 dx \]

\[ T_s = \frac{1}{2} m_s \dot{p}_{os}^T \dot{p}_{os} + \frac{1}{2} J_s \dot{\theta}_s^2 \]

\[ T_i = \frac{1}{2} m_i \dot{p}_{oi}^T \dot{p}_{oi} + \frac{1}{2} J_i \omega_i^2 \]

where \( \omega_{oi} \) is angular velocity of \( i \)-th frame expressed in \{O\} and \( \phi_i'(l_b) = d\phi_i(x)/dx \) evaluated at \( x = l_b \). In addition, we can calculate \( \alpha_{ij} \) and \( b_{ij} \) from the mode shapes determined in Sec. II-B.1.
In addition, potential energy can be similarly defined as follow
\[ U = U_{b_c} + U_{b_s} + U_s + \sum_{i=1}^{l_3} U_i \]
where \( U_{b_c} \) and \( U_{b_s} \) are elastic and gravitational energy of the beam, respectively. The other terms follow the same subscripts in the kinetic energy in the above equation. Then, we can express each potential energy term as
\[ U_{b_c} = \frac{EI}{2} \int_0^l \left( \frac{d^2w(x)}{dx^2} \right)^2 \, dx = \frac{EI}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} c_{ij} \delta_i(t) \delta_j(t) \]
\[ U_{b_s} = \rho A g^T \int_0^l \rho \phi_i \phi_j \, dx = - (\rho Al_b) g \left( \frac{1}{2} b \right) \]
\[ U_s = m_s g^T p_{poi}, \quad U_i = m_i g^T p_{poi} \]
where \( g = [-g; 0] \in \mathbb{R}^2 \) is gravitational acceleration and, we have the following equations from the orthogonality property.
\[ c_{ij} = \int_0^l \phi_i \phi_j \, dx = \begin{cases} 0 & \text{if } i \neq j \\ \omega_i^2 / EI & \text{if } i = j \end{cases} \]

Now we define generalized coordinate as \( \varphi := [q_r; q_f] \in \mathbb{R}^7 \) where \( q_r = [q_1; q_2; q_1] \) and \( q_f = [q_3; q_2; q_3] \). With the definition of the generalized coordinate and the Lagrangian \( L = T - U \) of the system, we can obtain the dynamics as following
\[
\begin{bmatrix} M_{rr} & M_{rf} \\ M_{fr} & M_{ff} \end{bmatrix} \dot{\begin{bmatrix} \dot{q}_r \\ \dot{q}_f \end{bmatrix}} + \begin{bmatrix} C_{rr} & C_{rf} \\ C_{fr} & C_{ff} \end{bmatrix} \begin{bmatrix} \dot{q}_r \\ \dot{q}_f \end{bmatrix} + \begin{bmatrix} g_r \\ g_f \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} K \begin{bmatrix} q_r \\ q_f \end{bmatrix} = \begin{bmatrix} \tau_r \\ \tau_f \end{bmatrix} \]
where \( K = \text{diag}(0; c_1; c_2; \cdots; c_3) \in \mathbb{R}^{4 \times 4} \) is structural spring which can be simplified by the orthogonal property, \( \tau_r = [\tau_3; \tau_2; \tau_1] \in \mathbb{R}^{3}, \) and \( \tau_f = [\tau_2; 0; 0; 0] \in \mathbb{R}^{4}. \)

### III. PASSIVE DECOMPOSITION AND CONTROL DESIGN

In this section, we design two control laws: 1) vibration-suppression control in the stage-vibration dynamics and 2) joint tracking control in its orthogonal complement. To achieve two different control objectives separately, we first deploy passive decomposition [3] to decompose the dynamics into the stage-vibration dynamics and its orthogonal complement. Due to the dynamically decomposed stage-beam dynamics, we can independently design the vibration suppression control by only using the stage motion. For the vibration suppression control, we use the LQR control based on the controllability analysis of the linearized stage-beam dynamics. Finally, we design the joint tracking control and show the tracking error convergence if the vibration is sufficiently suppressed.

#### A. Passive Decomposition

In this paper, what we want is to suppress the vibration of the flexible beam by using the stage motion while the manipulator follows the desired trajectory. However, due to the coupling terms in the inertia matrix, i.e. dynamic coupling, it is hard to separately design each control. To address this problem, we utilize passive decomposition [3] to decompose the dynamics (8) into stage-vibration dynamics and its orthogonal complement. We also show that if the vibration is suppressed, then the orthogonal complement dynamics is equivalent to the pure manipulator dynamics.

Following [3], let us first define the coordination map \( h(q) = q_f \) which is the generalized coordinate of the stage and the beam. Then, we can split the tangent space of the system s.t.,
\[ \Delta^\top := \{ \dot{q} \in \mathbb{R}^n | \mathcal{L}_q h(q) = \mathcal{L}_q q_f = 0 \} = \{ \partial \mathcal{L}_q / \partial q \} \]
\[ \Delta^\perp := \{ v \in \mathbb{R}^n | v^T M(q) \xi = 0, \forall \xi \in \Delta^\top \} \]
where \( \mathcal{L}_q \) is the Lie derivative of \( h(q) \) along \( \dot{q} \). This then implies that the tangent space of the system splits s.t.,
\[ T_q \mathcal{M} = \Delta^\top \oplus \Delta^\perp \]
where 1) \( \Delta^\top \) is called tangent distribution (i.e., parallel to the level set of \( h(q) \)) and 2) \( \Delta^\perp \) is called normal distribution (i.e., orthogonal complement of \( \Delta^\top \) w.r.t. the inertia matrix \( M(q) \)). See [3] for more details. We call the dynamics projected on the tangent distribution locked system and the system on \( \Delta^\perp \) shape system.

From the above definition of each distribution, we can write the velocity of the system
\[ \dot{q} = [\Delta^\top \Delta^\perp] \begin{bmatrix} v_{E} \\ v_{L} \end{bmatrix} = \begin{bmatrix} I_3 & S_E \\ 0_{4 \times 3} & I_4 \end{bmatrix} \begin{bmatrix} v_{E} \\ v_{L} \end{bmatrix} =: \nu \]
where \( \Delta^\top = [I_3; 0_{4 \times 3}] \in \mathbb{R}^{7 \times 3} \) and \( \Delta^\perp = [S_E; I_4] \in \mathbb{R}^{7 \times 4} \) are matrices identifying \( \Delta^\top \) and \( \Delta^\perp \), respectively. \( v_L \in \mathbb{R}^4 \) is the locked system motion and \( v_{E} \in \mathbb{R}^4 \) is the shape system motion which is same as the motion of the stage-beam, i.e. \( \dot{q}_2 \). We can find \( S_E = -M^{-1}_{rr} M_{rf} \in \mathbb{R}^{3 \times 4} \) from the relation \( \Delta^\top M(q) \Delta^\perp = 0 \).

By multiplying \( S^T \) to equation (8) with the relations \( \dot{q} = S\nu \) and \( \dot{q} = \dot{S}\nu + \dot{S} \nu \), we can obtain the decomposed dynamics given by
\[
\begin{bmatrix} M_L & 0 \\ 0 & M_E \end{bmatrix} \begin{bmatrix} \dot{v}_L \\ \dot{v}_E \end{bmatrix} + \begin{bmatrix} C_L & C_{LE} \\ C_{EL} & C_E \end{bmatrix} \begin{bmatrix} v_L \\ v_{E} \end{bmatrix} + \begin{bmatrix} g_L \\ g_E \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} K \begin{bmatrix} q_r \\ q_f \end{bmatrix} = \begin{bmatrix} \tau_L \\ \tau_E \end{bmatrix} \]
(9)
where \( M_L = M_{rr}, M_E = S^T_E M_{rf} + M_{ff}, \)
\[ \begin{bmatrix} g_L \\ g_E \end{bmatrix} = \begin{bmatrix} S^T_E \dot{g}_r + g_f \\ \tau_L + \tau_f \end{bmatrix} = \begin{bmatrix} S^T_E \tau_r + \tau_f \end{bmatrix} \]
The Coriolis terms is calculated by \( S^T(M(q) \dot{S} + C(q, \dot{q})S) \) where \( C(q, \dot{q}) \) is the Coriolis matrix in (8).

The shape system still describes the dynamics of the stage and vibrational motion which we can express as
\[ M_E \ddot{q}_f + C_E \dot{q}_f + K q_f + g_f = \tau_f + f_n \]
(10)
where \( f_n := -C_{EL} v_L + S^T_E (\tau_r - g_r) \) is the coupling force, and thus, we could interpret dynamics above by the mass-spring-damping system of \( q_f \) with the gravity \( g_f \) excited by the stage motion and the coupling force.
If the shape system is stabilized, i.e. \( \dot{q}_f \to 0 \), then the locked system converges to the pure manipulator dynamics as follows

\[
M_{rr} \ddot{q}_r + C_{rr} \dot{q}_r + g_r = \tau_r
\]

Thus, one possible approach to achieve two different control objectives, vibration suppression and tracking control, is to use the stage motion to subdue the vibration in the shape system and to design the tracking control in the locked system.

B. Vibration Suppression Control Design

We aim to stabilize the stage-beam dynamics with only the stage control input \( \tau_s \in \mathbb{R} \) regardless of the motion of the manipulator. However, since the stage-beam system is underactuated, i.e. one DOF stage actuation and four DOF stage-beam states, we cannot easily stabilize the stage-beam system through \( \tau_s \), and should examine the controllability of the system.

For the controllability analysis, let us define the state \( x = [x_1; x_2] \in \mathbb{R}^8 \) where \( x_1 = q_f \) and \( x_2 = \dot{q}_f \), and suppose that the manipulator is moving slowly enough to enable linearization of the shape system (10) around the equilibrium point with a fixed manipulator configuration. From the shape system dynamics, the equilibrium points can be found as

\[
\begin{align*}
\dot{x}_s^* &= 0 \quad (11) \\
\ddot{x}_s^* &= \dot{K}^{-1}(B^T \tau_1 - g_E) \quad (12)
\end{align*}
\]

where \( \dot{K} = \text{diag}([k_s; c_{11}; c_{22}; c_{33}]) \) is the augmented stiffness with \( k_s \). This augmented stiffness comes from the design of the stage input expressed by

\[
\tau_s = -k_s q_s - b_s \dot{q}_s + u_s \quad (13)
\]

where the first two terms are for stabilization of the stage since the stage must be stable with the vibration suppressed. The last term \( u_s \) is the auxiliary control input for the vibration suppression. The second equation (12) presents some offset deformation induced by gravity force and manipulator torque at a certain configuration of the manipulator.

Linearization of (10) at the equilibrium point gives the blocked matrices system

\[
\dot{x} = F \ddot{x} + G u_s = \begin{bmatrix}
F_1 & I_4 \\
F_2 & G_1
\end{bmatrix} \begin{bmatrix}
\ddot{x}_1 \\
\ddot{x}_2
\end{bmatrix} + \begin{bmatrix}
0_{4 \times 1} \\
I_{3 \times 3}
\end{bmatrix} u_s \quad (14)
\]

where \( \ddot{x}_i = x_i - x_i^* \). The system matrices \( F_1 \) and \( F_2 \in \mathbb{R}^{4 \times 4} \) and input distribution matrix \( G_1 \in \mathbb{R}^4 \) can be clearly expressed by

\[
\begin{align*}
F_1 &= -M_E^{-1} \left( \frac{\partial g_E}{\partial q_f} + \dot{K} \right), \\
F_2 &= -M_E^{-1} B \\
G_1 &= M_E^{-1} e_1
\end{align*}
\]

where \( e_1 = [1; 0; 0; 0] \) and \( B = \text{diag}([b_s; 0; 0; 0]) \) is the damping matrix. All matrices and vectors are evaluated at the equilibrium point obtained from (11) and (12).

We then numerically analyze the controllability of the linearized stage-beam system for all manipulator’s configuration, and find that the linearized system in which the control input is the only stage motion is controllable for every possible configuration of the manipulator. The fact that the configurations of robotic system is not able to affect the controllability seems reasonable because configurations mostly influence on the change of the gravity vector \( g_E \). When the manipulator is stretched out, the gravity term is equal to zero (See (a) in Fig. 5). Even in this case, the linearized system is controllable due to \(-M_E^{-1}K\) term in \( F_1 \).

Based on the controllability analysis, the LQR control can be used for the stage input to suppress the vibration

\[
u_s = -K_{LQR} \dot{x} \quad (15)\]

where \( K_{LQR} \in \mathbb{R}^{1 \times 8} \) is the LQR gain and \( \dot{x} = [\dot{x}_1; \dot{x}_2] \). Therefore, by combining (13) and (15), total input of the stage is given by

\[
\tau_s = -k_s q_s - b_s \dot{q}_s - K_{LQR} \dot{x} \quad (16)
\]

The controllability analysis and LQR control design are based on slower operator behavior assumptions, i.e., \( \dot{q}_r \approx 0 \), but our approach has been shown to work properly as observed in the experiments in Sec. IV.

C. Joint Tracking Control Design

To attain the joint tracking control, consider the locked system:

\[
M_L \ddot{q}_L + C_L \dot{q}_L + C_L E \dot{q}_f + g_L = \tau_L \quad (17)
\]

where again \( \dot{q}_L = \dot{q}_r - S_E \dot{q}_f \in \mathbb{R}^3 \). Since the locked system is fully-actuated, we can easily design the tracking control s.t.,

\[
\tau_L = f^p_L + M_L \ddot{q}_L - B_L \dot{q}_L - K_L e_L \quad (18)
\]

where \( f^p_L = C_L (\dot{q}_r^d - S_E \dot{q}_f^d) \) and \( C_L \dot{q}_f^d + g_L \) is the cancel out terms, \( e_L = q_r - q_r^d \) is the tracking error, and \( \dot{q}_f^d \) is the desired joint trajectory. Here, for the control design, we use the passivity property of the dynamics, i.e. skew symmetric property of \( M_L - 2C_L \). Then, it is possible to obtain the closed-loop dynamics by

\[
M_L \ddot{q}_L + (C_L + B_L) \dot{q}_L + K_L e_L = M_L \dot{q}_f^d + B \dot{q}_f^d
\]

which converges to

\[
M_L \ddot{q}_L + (C_L + B_L) \dot{q}_L + K_L e_L = 0 \quad (19)
\]

as the vibration is suppressed as obtained in Sec. III-B.

**Theorem 1** Consider the linearized shape system (14) with the vibration suppression control (16) and the locked system (17) with the controls (13) and (18). Then the following are true:

1) vibration is suppressed, i.e. \( \dot{q}_f \to 0 \)
2) manipulator follows the desired trajectory, i.e. \( (q_r - q_r^d) \to 0 \)
A. Test Setup

The system consists of three DOF manipulator, one DOF linear stage, and a vertical flexible beam. See Fig. 4. First of all, we have chosen the Dynamixel Pro of Robotics for the motors of the manipulator because of its compact size, acceptable torque level, and torque-controllability. The resolution of encoders is 501,900 or 304,000 p/rev according to the model of the motors. The Dynamixel Pro supports torque control mode and the update rate is around 400Hz with three motors. For accurate actuate, we calibrate the command-output force relation by using force sensor. Moreover, since the motors has high gear-ratio (for instance, the HS4-200 model used for the $\theta_1$ has 500:1 ratio), we compensates the motor (static) friction for better performance.

Next, for the linear stage, we use SMC-LEFS series which is ball-screw type linear actuator driven by 200W AC servo motor. The lead of the stage is chosen as 24mm which is the longest option of the model for the fast motion. The resolution of the stage encoder is 18 bit, i.e. 262,144 p/rev. We also calibrate by using force sensor to find the relation between the analog input and the output linear force. The update rate of the stage is around 1KHz.

Aluminium bars are used for the flexible beam. The length $l_b$ is 0.907m, thickness is 0.012m, and the width is 0.05m. For the calculation of the natural frequencies described in Sec. II-B.2, we use the encoders to measure the motions of the manipulator and stage system. And Optitrack, which is a motion capture system, is applied to measure the beam deflections.

B. Joint Tracking and Vibration Suppression Experiment

To verify the performance of the proposed controllers (16) and (18), we perform joint tracking control experiments with large enough initial joint error to see the performance of the vibration suppression control against dynamics coupling effect. At the beginning, the manipulator starts with large enough configuration error and then converges to the desired configuration as shown in (a) and (b) in Fig. 5. The convergence motion of the manipulator causes the system oscillation due to the dynamics effect. If we apply the vibration suppression control, then the system quickly becomes stable. Otherwise, it takes much longer time to be stabilized. Furthermore, around 15 sec. and 30 sec. we push the system to see the robustness and the performance against the unmodeled external disturbance. See (d) in Fig. 5.

Fig. 6 clearly presents the performance of the vibration suppression control. If we do not apply the vibration suppression control, then the vibration caused by both manipulator motion, i.e. dynamics effect, or the external force is attenuated by the structural damping which is not modeled in this paper. Since the unmodeled structural damping is small, it takes long time (more than 30 sec.) to stabilize the system.

On contrary to this, if we apply the vibration suppression control, the vibration is actively subdued in 5 sec.

Proof: Let us first consider the shape system dynamics (10) and the stage input (13). Then, as explained in Sec. III-B, the linearized system of the shape system dynamics becomes

$$\dot{x} = F\dot{x} + Gu_a$$

Based on the controllability analysis in Sec. III-B, we can design the stage motion controller $u_a$ as defined in (15) such that the closed-loop system $\dot{x} = (F - GK_{LQR})\dot{x}$ is stable, i.e. $\dot{x} \to 0$. Thus, we can conclude that $q_f \to q_f^*$ and $\dot{q}_f \to 0$, i.e. the vibration is suppressed.

Next, let us define the Lyapunove function candidate as

$$W = \frac{1}{2} \dot{e}_L^T M_L \dot{e}_L + \frac{1}{2} \dot{e}_L^T K_L \dot{e}_L$$

where again $e_L = q_r - q_L^*$, $M_L$ is the inertia matrix of the locked system, and $K_L$ is the position feedback gain matrix in (18). Then we obtain the time derivative of $W$

$$\dot{W} = \dot{\dot{e}}_L^T M_L \dot{e}_L + \frac{1}{2} \dot{e}_L^T \dot{M}_L \dot{e}_L + \dot{\dot{e}}_L^T K_L \dot{e}_L$$

$$= -\dot{\dot{e}}_L^T B_L \dot{e}_L + \dot{\dot{e}}_L^T M_L \dot{e}_L \left( B_2 \dot{q}_2 + B_2 \dot{q}_2^* \right)$$

For the second line, we use the passivity property, that is, $M_L - 2C_L$ is skew symmetric. Furthermore, if the vibration is suppressed and the stage is stabilized so that $\dot{q}_f = \ddot{q}_f \approx 0$, then we obtain $\dot{W} = -\dot{\dot{e}}_L^T B_L \dot{e}_L$.

Then, according to Barbalat’s lemma with the closed-loop dynamics (19), we can conclude that the equilibrium $\dot{e}_L = e_L = 0$ is globally asymptotically stable.

IV. EXPERIMENTS

We conduct experiments to verify the performance of the proposed controllers designed in Sec. III-B and III-C. The performances of the trajectory tracking and vibration suppression are provided. During the tracking, we excite the system to show the robustness of the proposed controllers.
V. CONCLUSION

In this paper, we proposed a novel control framework of a manipulator-stage system supported by a vertical flexible beam. The dynamics modelling of the system is developed by using the Euler-Bernoulli theory and Euler-Lagrange equation. Then, the use of the passive decomposition splits the dynamics into the stage-beam dynamics and its orthogonal complement. By doing that, we can separately design two different controller: 1) the vibration suppression control with the stage motion; and 2) the joint tracking control of the manipulator. We also achieved the tracking with the vibration subdued. The experiment results show the performance of the proposed control framework and robustness against external excitation.

REFERENCES