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Agreement with Non-Uniform Information Delays

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***Abstract*— We propose a novel agreement framework for multiple (possibly heterogeneous) agents evolving on a directed information graph with non-uniform delays. Our proposed framework can ensure agreement of a certain scalar quantity among the agents, as long as 1) for each agent, we can design a local control s.t. its closed-loop transfer function has unit gain at dc and gain strictly less than unity elsewhere; 2) the information graph has a globally reachable node (i.e. there exists a path from it to every other nodes); and 3) the information delays are finite constants. Rendezvous simulation is performed to verify the theory.**

***Index Terms*— cooperative control, multiagent agreement, information graph, information delays, spectral radius theorem.**

I. Introduction

Recently, multiagent cooperative control has drawn substantial research effort from many researchers. This is in part because it would enable us to realize many powerful engineering/computer applications (e.g. mobile sensor networks and distributed robotic surveillance/rescue [1], [2], [3], distributed data processing [4]), as well as to understand many fascinating phenomenon in nature and society (e.g. schooling of fish [5], human collective behaviour [6]). See [7] for a collection of such research efforts.

One of the primary goals of this multiagent cooperative control is how to achieve agreement among the agents, that is, if we have n-agents, we want $x_i(t) \rightarrow x_k(t)$ (i.e. $x_i(t) - x_k(t) \rightarrow 0$), $\forall i, k \in \{1, 2, ..., n\}$, where $x_i(t)$ is a variable of interest for the agent i. Depending on the applications, this variable $x_i(t)$ could be position (i.e. rendezvous [8]), position (with some offset) and velocity (i.e. flocking [9], [10]), or orbit and orbiting speed (i.e. cyclic pursuit [11]).

Compared to conventional control problems, one of unique technical challenges of this multiagent agreement problem is how to analyze the effects of information flow topology among the agents. This is important, because it determines how the local action of each individual agent propagates throughout the group. However, at the same time, only certain information topologies would be feasible other than the fully-connected one, especially when the number of agents is large. Moreover, different than conventional problems where the inter-agent interaction is often two-way (e.g. Newton's third law), information flow in the multiagent system can be just one-way. Thus, a new approach is necessary.

Another important challenge of the multiagent agreement problem is possible time delays in the inter-agent information flows. Such delays can occur, if the agents are remotely located so that the signal transmission takes non-negligible time (e.g. acoustic wave communication among underwater vehicles [12]), or if their communication medium is unreliable (e.g. Internet). This information delay can even result in unstable group behavior. Thus, it is important to properly analyze their effects on the group behavior. Compared to conventional control problems, this delay problem in the multiagent system is more difficult and complicated, as its effect is coupled with the characteristics of the inter-agent information flow topology.

For analyzing the effects of the inter-agent information flow topology, algebraic graph theory has been successfully applied and numerous strong results based on that have been reported (e.g. [8]-[10], [12]-[17]), where the inter-agent information flow topology is represented by the information graph, whose nodes and edges are corresponding to the agents and information flows between them, respectively.

In contrast, for the delay problem, currently available results are relatively rare and often applicable only to fairly restrictive settings. In [12], [13], uniform constant delay is considered, that is, all the information delays should be the same. The schemes proposed in [18], [19] can deal with non-uniform constant delays. However, they require the information graph to be (at least) balanced. This implies that, for each agent, number of agents to which it sends information should be always the same as that from which it receives information. General information graph with non-uniform time-varying delays are considered in [20], [21], where agents are modeled as first-order discrete-time systems. However, it is not easy to see how the techniques used there can be applied when the agents have continuous-time dynamics. Although this list is by no means exhaustive, we believe that it adequately covers the status quo research vista in this direction.

In this paper, we present a novel agreement framework for multiple (possibly heterogenous) continuous-time agents evolving on a (fixed) directed information graph with non-uniform constant delays.

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We first design a local control for each agent s.t. its closed-loop transfer function has unit gain at zero frequency (i.e. dc) and strictly less than unity elsewhere. This would be possible for many practical systems (e.g. feedback-linearizable nonlinear systems, controllable and observable linear time-invariant systems). Then, using the non-expansiveness of the constant delay operator (i.e. unit gain in the frequency domain) and the spectral radius theorem [22], we show that the closed-loop dynamics of the total group is marginally stable with the marginality (i.e. sustained oscillation with a constant amplitude) occurring only at dc. In other words, even with non-uniform constant delays, all the signals with non-zero frequency will die out, while the group's steady-state converges to a (generally non-zero) dc-offset. Using algebraic graph theory, we can further show that this steady-state dc-offset ensures agreement among the agents, if and only if their information graph has a globally reachable node [14].

It is also noteworthy that our proposed framework can address a group of heterogeneous agents, as long as we can find a local control for each agent s.t. its closed-loop dynamics has the aforementioned frequency response characteristics. Along with the inter-agent information delays, this heterogeneousness among the agents has been pointed out as an important future research topic in a recent survey paper [23].

The rest of this paper is organized as follows. In Sec. II, the information graph and some results of the graph theory are introduced. In Sec. III, we present our main result, a novel linear agreement protocol, which ensures multiagent agreement even when the inter-agent information is subject to non-uniform constant delays. In Sec. IV, we apply this designed agreement protocol to the multiagent rendezvous problem [8] (i.e. position coordination of mobile agents) and perform a simulation. Some concluding remarks and future research directions are then given in Sec. V.

II. Information Graph

Consider n-agents. Then, the information topology among them can be represented by their (unweighted) directed graph $G_u := \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} := \{v_1, ..., v_n\}$ is a set of nodes (i.e. agents), and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges (i.e. ordered pairs of the nodes). See Fig. 1 for some examples. For notational convenience, we exclude self-joining edges out from $\mathcal{E}$, i.e. $e_{ii} \notin \mathcal{E}$, $\forall i \in \{1, ..., n\}$. When $e_{ik} := (v_i, v_k) \in \mathcal{E}$, we call v_i and v_k the head and tail of the edge e_{ik}, respectively. This would imply that the information is flowing from v_k to v_i. For instance, this can happen if v_i sense the state of v_k or v_k sends its state information to v_i. Let us also define the information neighbor of v_i s.t.

$$\mathcal{N}_i := \{k \mid e_{ik} = (v_i, v_k) \in \mathcal{E}\} \tag{1}$$

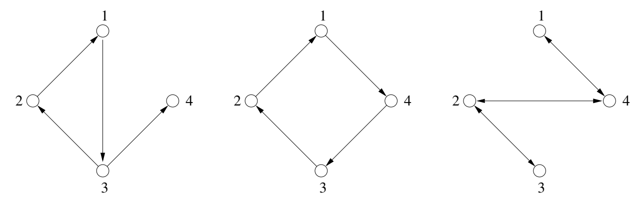

Fig. 1. Examples of information graphs: the first graph has v_1, v_2 and v_3 as globally reachable nodes, while the second and third are strongly-connected cyclic and undirected graphs with all the nodes being globally reachable.

i.e. the set of all the tails of v_i.

In many practical applications, inter-agent information flow (i.e. each edge of G_u) may be subject to time-delays (e.g. for remotely-located agents) and non-uniform reliability (e.g. different signal-to-noise ratio). To address these aspects, we define 1) a delay map $\mathcal{T} : \mathcal{E} \to \Re^+$, s.t. for $e_{ik} \in \mathcal{E}$, $\mathcal{T}(e_{ik}) = \tau_{ik}$, where $\tau_{ik} \geq 0$ is a finite constant delay; and 2) a weight map $\mathcal{W} : \mathcal{E} \to \Re^+$ assigning a (positive) weight to each edge s.t. if $e_{ik} \in \mathcal{E}$, $\mathcal{W}(e_{ik}) = \alpha_{ik}$, where $\alpha_{ik} > 0$ and $\sum_{k \in \mathcal{N}_i} \alpha_{ik} = 1$. Then, the information flow topology and characteristics among the n-agents can be completely represented by the (directed, weighted, and delayed) graph $G := \{G_u, \mathcal{W}, \mathcal{T}\} = \{\mathcal{V}, \mathcal{E}, \mathcal{W}, \mathcal{T}\}$. We call such a graph G *information graph* of the agents.

We say that a node $v_i \in \mathcal{V}$ is a globally reachable node [14], if there exists a path (i.e. a combination of edges in $\mathcal{E}$) from it to every other nodes in G. We also say that G is strongly connected, if, for every two nodes v_i, v_k, there exists a path connecting them. Thus, every node of a strongly connected graph is globally reachable.

If the communication delays τ_{ik} are all zeros or negligible, the information graph G becomes a usual directed and weighted graph $G_w := \{\mathcal{V}, \mathcal{E}, \mathcal{W}\}$. Then, the (normalized) adjacency matrix $A(G_w) \in \Re^{n \times n}$ of G_w is defined by

$$A_{ik}(G_w) = \begin{cases} \alpha_{ik} & \text{if } e_{ik} = (v_i, v_k) \in \mathcal{E} \\ 0 & \text{otherwise} \end{cases}. \tag{2}$$

Note that all the diagonals of $A(G_w)$ are zeros, since $e_{ii} \notin \mathcal{E}$. This adjacency matrix $A(G_w)$ has the following properties: 1) all of its eigenvalues have modulus less than or equal to unity, that is, they are residing in/on the unit circle centered at origin in the complex plane (from Grešgorin's disk theorem [24]); 2) 1 is always an eigenvalue of $A(G_w)$ with $u := [1, 1, .., 1]^T \in \Re^n$ being its corresponding eigenvector (i.e. $A(G_w)u = u$), because $\sum_{k=1}^{n} A_{ik}(G_w) = 1$ (from the fact that $\sum_{k \in \mathcal{N}_i} \alpha_{ik} = 1$); and 3) this 1 is a simple eigenvalue of $A(G_w)$ if and only if G has a globally reachable node [14], [15].

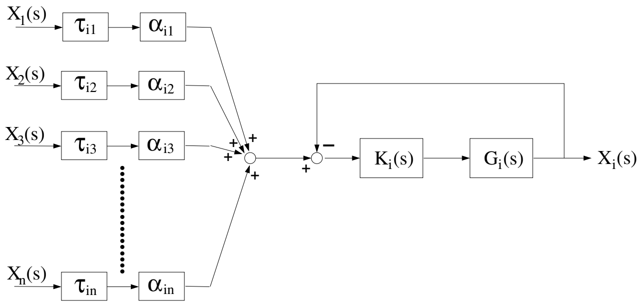

Fig. 2. Linear agreement protocol: $G_i(s)$ and $K_i(s)$ are the underlying open-loop dynamics and the local agreement protocol for the agent i, and τ_{ik} and α_{ik} are the delay and weight on the link $e_{ik} = (v_i, v_j)$. Here, we set $\alpha_{ik} = 0$, if e_{ik} is not in $\mathcal{E}$.

III. Linear Agreement Protocol Design

Now, suppose that each agent has its own (scalar) variable of interest. We denote this variable by $x_i(t)$ for the agent i. Then, we say that the agents reach agreement if $x_i(t) \to x_k(t)$ as $t \to \infty$, for every $i, k \in \{1, 2, ..., n\}$.

To achieve this, we propose the linear agreement architecture as shown in Fig. 2, where $G_i(s)$ and $K_i(s)$ are the Laplace transforms of the underlying open-loop dynamics and the local agreement control for the agent i, respectively. Also, τ_{ik} and α_{ik} are the delay and weighting factor for the communication link e_{ik}. In Fig. 2, if $e_{ik} \notin \mathcal{E}$ (i.e. link e_{ik} is disconnected), we set $\alpha_{ik} = 0$. Then, we can show that the closed-dynamics for the i-th agent is given by

$$X_i(s) = H_i(s) \sum_{k \in \mathcal{N}_i} \alpha_{ik} e^{s\tau_{ik}} X_k(s) \tag{3}$$

where $\mathcal{N}_i$ is the information neighbor of the agent i defined in (1), s is the Laplace variable, $X_i(s)$ is the Laplace transform of $x_i(t)$, and $H_i(s)$ is the closed-loop dynamics of the agent i given by

$$H_i(s) := \frac{G_i(s)K_i(s)}{1 + G_i(s)K_i(s)}. \tag{4}$$

Here, we assume that this $H_i(s)$ is strictly stable with no unstable pole-zero cancellation, $\forall i \in \{1, 2, ..., n\}$.

Then, by stacking up the individual agent's dynamics (3), we can achieve the following closed-loop dynamics of the total group:

$$X(s) = H(s)X(s) \tag{5}$$

where $X(s) := [X_1(s), X_2(s), ..., X_n(s)]^T \in \mathrm{C}^n$, and $H(s) \in \mathrm{C}^{n \times n}$ is defined by

$$H_{ik}(s) := \begin{cases} \alpha_{ik} e^{s\tau_{ik}} H_i(s) & \text{if } e_{ik} = (v_i, v_k) \in \mathcal{E} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where $H_{ik}(s)$ is the ik^{th} component of $H(s)$, $i, k \in \{1, 2, ..., n\}$. Here, note that the diagonals of $H(s)$ are all zeros, since $e_{ii} \notin \mathcal{E}$. This closed-loop group dynamics (5) forms the self-joining feedback-loop as shown in Fig. 3. The following theorem is a direct consequence of the spectral radius stability theorem [22], the Geršgorin's disc theorem [24], the non-expansiveness of the constant delay operator, and the property of the adjacency matrix $A(G_w)$ in (2).

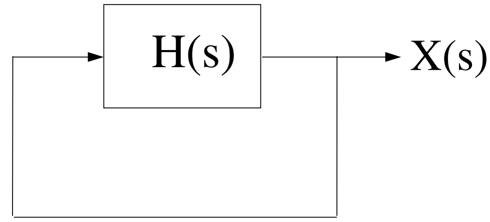

Fig. 3. Group dynamics (5) as a self-joining feedback-loop, where $H(s)$ contains the closed-loop dynamics $H_i(s)$, the weights α_{ik}, and the time-delays $e^{s\tau_{ik}}$ in (3).

Theorem 1 *Suppose that $H_i(s)$ in (4) possesses the following properties:*

$$|H_i(jw)| < 1, \quad \forall w \in (0, +\infty] \tag{7}$$

$$\lim_{w \to 0} H_i(jw) = 1 \tag{8}$$

$\forall i \in \{1, 2, ..., n\}$, where $j := \sqrt{-1}$. Then, there exists a (generally non-zero) finite constant $c \in \Re$ s.t. for any initial condition, $x_i(t) \to x_k(t) \to c$ $\forall i, k \in \{1, 2, ..., n\}$, if and only if the information graph G has a globally reachable node.

Proof: From its definition (6) and the Geršgorin's disc theorem [24, pp.344], all the eigenvalues of $H(jw)$ in (5) are located in or on the union of following discs in the complex plane C:

$$D_i(w) := \{z \in \mathrm{C} : |z| \leq \sum_{k \in \mathcal{N}_i} |\alpha_{ik} e^{jw\tau_{ik}} H_i(jw)|\} \tag{9}$$

where, using the conditions (7)-(8), the property of α_{ik} (i.e. $\alpha_{ik} > 0$ and $\sum_{k \in \mathcal{N}_i} \alpha_{ik} = 1$), and the fact that $|e^{jw\tau_{ik}}| = 1$, we can show that

$$\sum_{k \in \mathcal{N}_i} |\alpha_{ik} e^{jw\tau_{ik}} H_i(jw)| \leq \sum_{k \in \mathcal{N}_i} \alpha_{ik} \leq 1$$

with the equality holding only for $w = 0$.

Thus, the spectral radius of $H(jw)$ is strictly less than unity $\forall w \in (0, +\infty]$, and less than or equal to unity for $w = 0$. Following the spectral radius theorem [22, pp.149], this implies that the feedback-loop in Fig. 3 is marginally stable, with the marginal behavior (i.e. sustained oscillation of finite constant magnitude) possible only at $w = 0$. This further implies that, in the closed-loop group dynamics (5), any signals with non-zero frequency will die out, and only that with zero-frequency (i.e. dc-component) will remain.

Let us denote such a sustained steady-state dc-component for the agent i by a finite constant $\bar{x}_i \in \Re$. Then, from Fig.

2 with the condition (8), we can show that this dc-offset $\bar{x}_i$ should satisfy the following algebraic condition

$$\bar{x}_i = \sum_{k \in \mathcal{N}_i} \alpha_{ik} \bar{x}_k. \tag{10}$$

Then, by stacking up this condition (10) for all the agents, we achieve the following matrix condition

$$\bar{x} = A(G_w)\bar{x} \tag{11}$$

where $\bar{x} := [\bar{x}_1, \bar{x}_2, ..., \bar{x}_n]^T \in \Re^n$ and $A(G_w)$ is the adjacency matrix of the weighted graph $G_w := \{\mathcal{V}, \mathcal{E}, \mathcal{W}\}$ as given by (2). This condition (11) is satisfied by any solution for the steady-state dc-offset $\bar{x}_i$.

This matrix condition (11) is always feasible (i.e. a nontrivial solution $\bar{x}$ for (11) always exists), since, as stated in Sec. II, 1 is always an eigenvalue of the adjacency matrix $A(G_w)$. Furthermore, this eigenvalue 1 of $A(G_w)$ becomes simple and its associated eigenvector is uniquely given by $[1, 1, .., 1]^T$, if and only if the information graph G has a globally reachable node [14], [15]. Therefore, $\bar{x} = c[1, 1, ..., 1]^T$ and $x_i(t) \to x_k(t) \to c$ (i.e. dc agreement) $\forall i, k \in \{1, 2, ..., n\}$ from any initial conditions, if and only if G has a globally reachable node, where $c \in \Re$ is a (unspecified and generally non-zero) finite constant. This completes the proof. ∎

IV. Example: Multiagent Rendezvous

In this section, we apply the results derived in Sec. III to the rendezvous problem (i.e. position agreement) of n multiple mobile agents with the variable of interest $x_i(t) \in \Re$ for the agent i being its own position. We assume that each agent i has the usual fully-actuated point-mass dynamics (i.e. double-integrator dynamics)

$$m_i \ddot{x}_i(t) = f_i(t)$$

where $m_i > 0$ is the inertia and $f_i(t) \in \Re$ is the agreement control designed s.t.

$$f_i(t) := -b_i \dot{x}_i(t) - k_i(x_i(t) - \sum_{k \in \mathcal{N}_i} \alpha_{ik} x_k(t - \tau_{ik}))$$

with $b_i > 0$ and k_i being the local damping gain and the target-tracking gain, respectively. Then, the agent i has the following closed-loop dynamics

$$m_i \ddot{x}_i(t) + b_i \dot{x}_i(t) + k_i(x(t) - \sum_{k \in \mathcal{N}_i} \alpha_{ik} x_k(t - \tau_{ik})) = 0.$$

Therefore, for Fig. 2, we have $G_i(s) = 1/(m_i s^2 + b_i s)$, $K_i(s) = k_i$ and

$$H_i(s) = \frac{G_i(s)K_i(s)}{1 + G_i(s)K_i(s)} = \frac{k_i}{m_i s^2 + b_i s + k_i}.$$

We set $b_i \geq 2\sqrt{m_i k_i}$ so that, being critically-damped or over-damped, this $H_i(s)$ satisfies the conditions (7)-(8) in Theorem 1.

We performed a simulation using four of these agents with all different inertias. We also used the first information graph in Fig. 4 with the agents $1, 2$ and 3 being globally reachable nodes. We also imposed non-uniform constant information delays: $\tau_{12} = 0.1$sec, $\tau_{23} = 0.3$sec, $\tau_{31} = 0.6$sec, and $\tau_{43} = 0.15$sec. Simulation results are shown in Fig. 4, where it is clear that multiagent rendezvous is achieved over a directed information graph even with the non-uniform delays. We also achieve similar rendezvous results with the cyclic and undirected graphs in Fig. 1, but, for brevity, we omit their results here. In Fig. 4, the behavior of the agent 4 is slightly different than those of others, and we believe that this is due to the information flow topology: in the first graph of Fig. 1, the agents $1, 2$ and 3 are connected by a cycle, while the agent 4 is outside of it

To verify the applicability of the propose framework for heterogeneous agents, we replaced some agents in the previous simulation by agents of different types, whose open-loop dynamics is given by the following single integrator dynamics

$$\dot{x}_i(t) = f_i(t)$$

where $x_i(t) \in \Re$ is the position, and $f_i(t) \in \Re$ is the agreement control designed s.t.

$$f_i(t) := -k_i(x_i(t) - \sum_{k \in \mathcal{N}_i} \alpha_{ik} x_k(t - \tau_{ik}))$$

where $k_i > 0$ is the control gain. We call agents of this type "kinematic" agents, while those used in the previous simulation are called "dynamic" agents. Then, the closed-loop dynamics of this kinematic agent is given by

$$\dot{x}_i(t) + k_i(x_i(t) - \sum_{k \in \mathcal{N}_i} \alpha_{ik} x_k(t - \tau_{ik})) = 0.$$

Thus, for Fig. 2, we have $G_i(s) = 1/s$, $K_i(s) = k_i$ and

$$H_i(s) = \frac{G_i(s)K_i(s)}{1 + G_i(s)K_i(s)} = \frac{k_i}{s + k_i}.$$

It is easy to see that this $H_i(s)$ satisfies the conditions (7)-(8) in Theorem 1.

We substituted the dynamic agents 2 and 4 in the previous simulation by two kinematic agents. We used the second strongly-connected cyclic information graph in Fig. 1 and imposed on it non-uniform constant delays of magnitude similar to those in the previous simulation. Simulation results are presented in Fig. 5, which clearly shows that the proposed framework can ensure agreement even for heterogeneous agents with non-uniform information delays. In Fig. 5, the trajectories of the kinematic agents 2 and 4 exhibit sharper turns around $(x, y) = (-1.8, 1.5)$[m] and $(x, y) =$

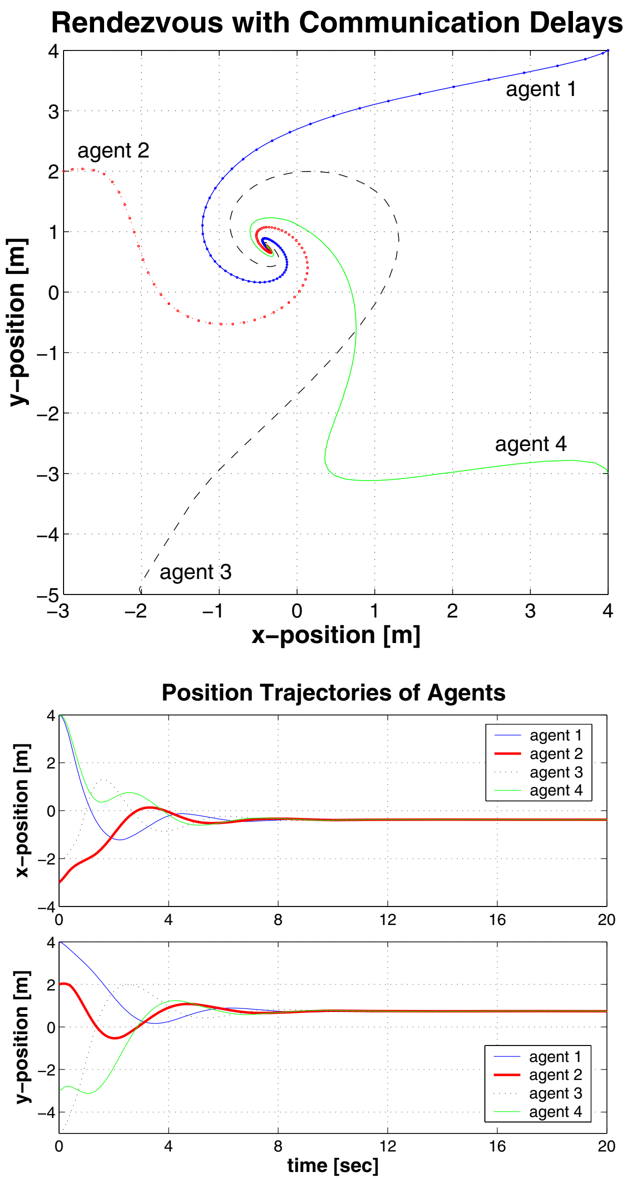

Fig. 4. Four homogeneous dynamic agents rendezvous with non-uniform time-delays.

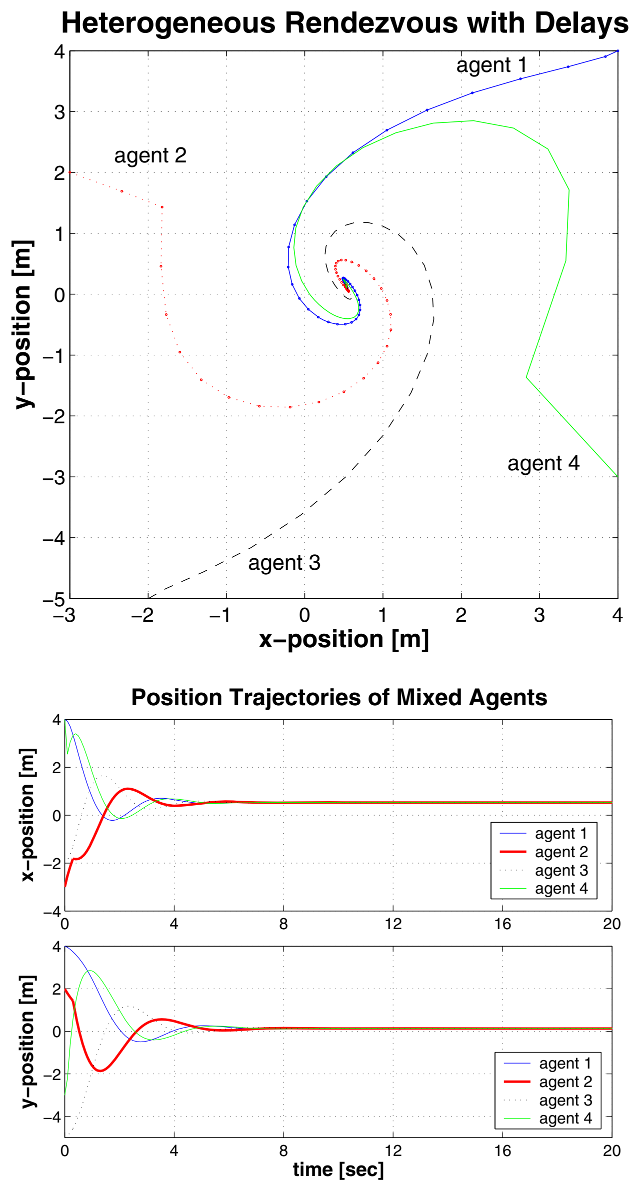

Fig. 5. Four heterogeneous agents rendezvous with non-uniform time-delays: agents $1, 3$ are the dynamic agents, while $2, 4$ are the kinematic agents.

$(2.8, -1.3)$[m], respectively. We think that this is because of 1) a sudden jump in the received information from an initial value (0 in this case) to a non-zero value when the real information is flowing in after the delay period passes; and 2) less system order of the kinematic agents, which make them more sensitive to the sudden jumps than the dynamics agents.

V. Conclusions

In this paper, we present a novel agreement framework for multiple agents, whose inter-agent information flow has a topology of a directed graph and is subject to non-uniform constant delays. We first design a local control s.t. the closed-loop dynamics of each agent has gain 1 at dc and strictly less than 1 elsewhere. Then, relying on the spectral radius theorem with the non-expansiveness of constant time-delay operator, we show that the steady-state of the closed-loop group dynamics converges to a dc-offset, which in turn guarantees the agreement among the agents, if and only if their information graph has a globally reachable node. To verify the theory, we perform a multiagent rendezvous simulation with homogeneous as well as heterogeneous agents.

Although our proposed framework is based on linear system theory, we believe that there would be a good possibility to extend it to more general nonlinear systems. This is partly because 1) the non-expansiveness of the constant-time delay operator can be directly carried over to the nonlinear domain;

and 2) the spectral radius theorem might be generalized to the small gain theorem, which is well-defined both in the linear and nonlinear system theories.

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