

## Preliminary Results on Quadrotor Manipulation Control

Hyunsoo Yang<sup>1</sup>, Dongjun Lee<sup>2</sup>

<sup>1</sup>School of Mechanical & Aerospace Engineering and IAMD, Seoul National University, Seoul 151-744, Korea  
 (E-mail: yangssoo@snu.ac.kr, djlee@snu.ac.kr)

**Abstract** - We propose controller for quadrotor with arm system. Quadrotor is underactuated which makes control problem difficult. Attached manipulator makes system dynamics complicate. To control end-effector position, we used passive decomposition and it provides decomposed dynamics. Using decomposed dynamics, we proposed backstepping-like controller to control end-effector position. Proposed controller achieve desired end-effector trajectory via underactuated system while moving manipulator. Simulation results are presented.

**Keywords** - quadrotor, manipulator, underactuation, cascaded control

### 1. Introduction

Unmanned aerial vehicle (UAV), especially quadrotor has been researched extensively in recent year [1-7]. Usual mobile robot have 2-dimensional mobility, however UAV extends its mobility to 3-dimensional space. Therefore, it is promising various applications: land scape survey, search and rescue, surveillance.

By attaching manipulator to quadrotor platform, its functionality is extended. Manipulator allows interaction with environment, therefore its application is not restricted to passive observation. As a result, quadrotor with manipulator is promising powerful applications than platform only: operation on high-rise building, assembly task using multiple robots.

However, there are only few research results about quadrotor with manipulator [1-7]. Manipulator makes dynamics complicatedly. Quadrotor and manipulator have coupled dynamics and also quadrotor is underactuated. These properties makes control problem difficult. There are some relevant works about quadrotor with some attached tool. Cable is used to grasp object [1], fixed tool is attached for tool operation such as screwdriver [2], gripper is studied which manipulation capability is restricted[3]. And there are some results about quadrotor with manipulator [4-7], but did not deal with full dynamcis [5], [6] and just deal with stabilization or preserve certain configuration[4], [6], [7].

In this paper, we apply passive decomposition [8] to deal with underactuation and its complex, coupled dynamics. Using passive decomposition, its dynamics is divided into underactuated and fully actuated dynamics. From these two dynamics, we proposed backstepping-like controller to control end-effector position. This preliminary result is restricted to xz plane motion.

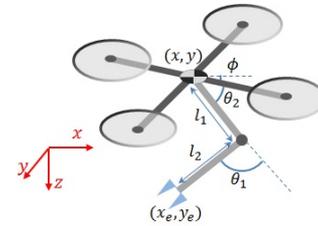


Fig. 1 Quadrotor with manipulator model.

## 2. Preliminary

### 2.1 Dynamics

Quadrotor with manipulator system is in the Fig.1. Manipulator has 2 revolute joint and 2 degree of freedom (DOF). From Fig.1, its configuration is defined as  $q = [x, z, \phi, \theta_1, \theta_2]^T$ , where  $(x, z)$  is  $x, z$  coordinate of geometric center of quadrotor in inertial frame,  $\phi$  is pitch angle and  $\theta_1, \theta_2$  are each joint angle.  $y$  is omitted, because we restrict its motion to  $xz$  plane quadrotor with manipulator can be written as Lagrangian dynamics.

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau$$

In this dynamics,  $\phi, \theta_1, \theta_2$  are fully actuated because it is directly controlled using its related input, however position dynamics is underactuated. This underactuation problem can makes difficult to control.

### 2.2 Passive Decomposition

Passive decomposition decomposes its original dynamics into two dynamics. Dynamics is decomposed using certain function. To divide its dynamics into fully actuated and underactuated part, we defined function as  $h(q) = [\phi, \theta_1, \theta_2]^T$ . Using  $h(q)$ , velocity space is divided into two subspaces

$$\Delta^\top := \{\dot{q} \in \mathbb{R}^5 | L_{\dot{q}}h(q) = 0\} = null\left(\frac{\partial h}{\partial q}\right) \quad (1)$$

$$\Delta^\perp := \{\dot{q} \in \mathbb{R}^5 | \dot{q}M(q)x, x \in \Delta^\top\} \quad (2)$$

where,  $\Delta^\top$  is locked system which does not affect  $h(q)$  and underactuated,  $\Delta^\perp$  is shape system which fully actuated. Above expression equally written as

$$\dot{q} = [\Delta^\top \ \Delta^\perp] \begin{pmatrix} \nu_L \\ \nu_E \end{pmatrix} \quad (3)$$

Using this divided velocity space, dynamics is decomposed into two dynamics equation which inherits passivity and Lagrangian dynamics like structure.

$$M_L \dot{\nu}_L + G_L = u_L \quad (4)$$

$$M_E \dot{\nu}_E + C_E \nu_E = u_E \quad (5)$$

Decomposed dynamics is decoupled in inertia matrix and especially represent center of mass dynamics. It also have decoupled between locked and shape system in coriolis term which is usually coupled (e.g.,  $C_{LE} \nu_E$ ,  $C_{EL} \nu_L$ )

### 3. Controller Design

We propose controller to control end-effector position. From kinematic equation, relation between center of mass and end-effector is defined as following

$$\begin{pmatrix} \dot{x}_e \\ \dot{z}_e \end{pmatrix} = \nu_L + B(r) + \dot{r} = -K \begin{pmatrix} x_e - x_e^d \\ z_e - z_e^d \end{pmatrix} + \begin{pmatrix} \dot{x}_e^d \\ \dot{z}_e^d \end{pmatrix} \quad (6)$$

where third term is desired relation which realize convergence to desired trajectory. Using this desired relation Eq. (1), desired  $\nu_L^d$  to achieve desired end-effector trajectory is given by

$$\nu_L = \left[ -K \begin{pmatrix} x_e - x_e^d \\ z_e - z_e^d \end{pmatrix} + \begin{pmatrix} \dot{x}_e^d \\ \dot{z}_e^d \end{pmatrix} - B(r)\dot{r} \right] \quad (7)$$

Equation (7) gives backstepping-like structure and lyapunov function is given by

$$V = \frac{1}{2} e_p^T e_p + \frac{1}{2\gamma} e_L^T M_L e_L \quad (8)$$

From lyapunov function, following equation guarantee exponential convergence of  $e_p$

$$u_L + M_L B(r) M_E^{-1} [u_E - C_E \dot{r}] = \gamma e_p - \alpha e_L + G_L + M_L [\ddot{p}_d - \lambda \dot{e}_p - \frac{dB}{dt} \dot{r}] \quad (9)$$

This is redundant control problem. We assign two control input  $u_L$  and  $u_E$  appropriately, then error is converge to zero and arm is also controlled to desired configuration. First, we assign  $u_L$  and get desired value of  $\phi$ . Next, the rest part of eq (9) is assigned by  $u_E$ . Using redundancy of  $u_E$ , we control  $\phi$  to  $\phi_d$ . Using this framework, quadrotor follows end-effector trajectory with error convergence.

Simulation result given by Fig. 2. Desired trajectory of end-effector is given. Manipulator is controlled to follow desired configuration. Although manipulator is moving and system is underactuated, end-effector follows desired trajectory well.

### 4. Conclusion

In this paper, we propose backstepping-like controller for underactuated quadrotor with manipulator system. Passive decomposition is used to decompose original dynamics into fully actuated and underactuated dynamics. To deal with underactuation problem, we propose backstepping-like controller. In simulation, proposed controller shows tracking performance of end-effector. Relevant future works are as following: 1) extending its workspace to 3D; 2) implementation in real platform.

### Acknowledgement

Research supported in part by the Basic Science Research Program(2012-R1A2A2A0-1015797) of the National Research Foundation (NRF) of Korea funded by the Ministry of Education, Science & Technology (MEST).

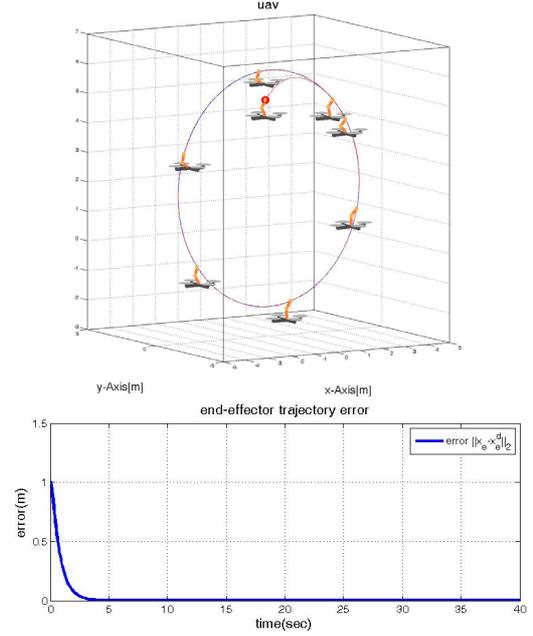


Fig. 2 Trajectory and error of end-effector.

### References

- [1] N. Michael, J. Fink, and V. Kumar. Cooperative manipulation and transportation with aerial robots. *Autonomous Robots*, 30(1):73–86, 2011.
- [2] D. J. Lee and C. Ha. Mechanics and control of quadrotors for tool operation. In *Proc. ASME Dynamics Systems and Control Conference*, pages 177–184, 2012.
- [3] D. Mellinger, Q. Linsey, M. Shomin, and V. Kumar. Design, modeling, estimation and control for aerial grasping and manipulation. In *Proc. IEEE Int'l Conference on Robotics and Automation*, pages 2668–2673, 2011.
- [4] X. Ding and Y. Yu. Motion planning and stabilization control of a multipropeller multifunction aerial robot. *IEEE/ASME Transactions on Mechatronics*, 18(2):645–656, 2013.
- [5] A.E. Jimenez-Cano, J. Martin, G. HereDia, and R. Cano A. Ollero. Control of an aerial robot with multi-link arm for assembly tasks. In *Proc. IEEE Int'l Conference on Robotics and Automation*, pages 4901–4906, 2013.
- [6] C. Korpela, M. Orsag, and P. Oh M. Pekala. Dynamic stability of a mobile manipulating unmanned aerial vehicle. In *Proc. IEEE Int'l Conference on Robotics and Automation*, pages 4907–4912, 2013.
- [7] V. Lippiello and F. Ruggiero. Exploiting redundancy in cartesian impedance control of uavs equipped with a robotic arm. In *Proc. IEEE Int'l Conference on Intelligent Robots and Systems*, pages 3768–3773, 2012.
- [8] D. J. Lee. Passive decomposition of mechanical systems with coordination requirements. *IEEE Transactions on Robotics*, 26(6):978–992, 2013.