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Hybrid PD-Based Control Framework for Passive Bilateral Teleoperation over the Internet ⋆

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Abstract: Let us consider the problem of bilateral teleoperation over the Internet. One of its aspects, which has not been much utilized so far, is its *hybrid* nature, that is, the (nonlinear) master and slave robots are continuous-time systems, while the Internet communication between them defines a discrete-time channel. Exploiting this hybrid nature and combining our prior results [Lee [2009], Huang and Lee [2010]], in this paper, we propose a novel hybrid proportional-derivative (PD) type passivity-enforcing bilateral teleoperation control framework over the Internet with such communication unreliability as varying-delay, packet-loss, data-duplication/swapping, etc. The closed-loop passivity of the teleoperator, position coordination and the force reflection between master and slave robots are mathematically proved.

Keywords: Internet teleoperation, passivity, hybrid system, varying delay, packet loss

1. INTRODUCTION

Let us consider a class of Internet bilateral teleoperation systems as shown in Fig. 1, which consist of (possibly nonlinear) continuous-time master and slave robots, sampled-data synchronization control (e.g. proportional derivative (PD) control), and discrete-time (imperfect) Internet communication. This (hybrid) teleoperation system is mechanically interacting with (generally unknown) human operators and slave environments, while allowing the human users to kinesthetically feel and interact with the slave environment. The primary goal is then how to ensure safety and stability of such mechanical interactions.

Energetic passivity has been widely used to achieve such safety and stability requirements [Anderson and Spong [1989], Lawrence [1993], Niemeyer and Slotine [2004], Lee and Spong [2006], Ryu and Preusche [2007]]. To explain this, let us consider the teleoperator as a two-port network (Fig. 2). The (closed-loop) teleoperator exchanges energy with the human and the environment through the corresponding power ports. If the (closed-loop) teleoperator and the human/environment are both energetically passive, then the total teleoperation system (including human/environment) in Fig. 1 is passive, implying robust stability. Passivity of the (closed-loop) bilateral teleoperator, however, is in general broken when the communication is imperfect (e.g. varying-delay, data-loss, packet-duplication/swapping). This is because such communication imperfectness can generate unwanted energy, which can turn the (closed-loop) teleoperator non-passive.

To address such imperfectness in the continuous-time communication channels, many techniques have been proposed, e.g.,: 1) for constant delay, [Anderson and Spong [1989]] initiated the scattering-based method, which was extended by [Niemeyer and Slotine [1991]] using the notion of wave variables; 2) assuming humans, environments and robots to be known linear time-invariant (LTI) systems, [Hu et al. [1995], Leung et al. [1995]] provide solutions based on H_∞-control and μ-synthesis; 3) [Lee and Spong [2006], Nuno et al. [2008]] showed that the simple PD control can enforce passivity of the closed-loop teleoperator, while also providing position/force coordination; and 4) for varying delay, [Lozano et al. [2002]] provided a solution by extending the scattering-based method. However, these results, derived for continuous-time communication channels, cannot handle with the general imperfectness of Internet communication (e.g., packet loss, data swap, etc) as well as the Internet's discrete-time nature.

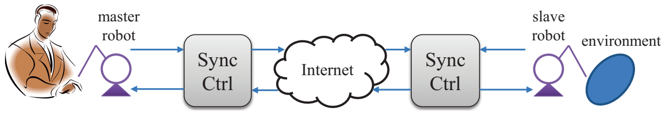

Fig. 1. Internet based bilateral teleoperation system.

For discrete (imperfect) Internet communication, much less results are available. [Stramigioli et al. [2005], Chopra et al. [2008]] extended the scattering-based method for the Internet teleoperation and could passively handle varying delay and packet loss. However, due to the lack of explicit position feedback, these scattering-based methods can suffer from the well-known problem of master-slave position drift, particularly in the presence of certain imperfectness of the Internet communication (e.g., packet loss).

In this paper, combining and extending our prior results of [Lee [2009], Huang and Lee [2010]], we propose a novel hybrid PD-type control framework, which, by exploiting the *hybrid nature* of the Internet teleoperation (i.e., discrete-time communication and continuous-time robots), can not only enforce the passivity of the closed-loop teleoperator, but also guarantee position/force coordination (in certain operating conditions), even with the imperfect Internet

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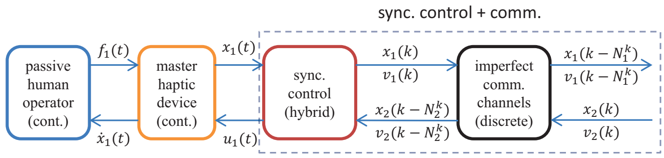

Fig. 2. Bilateral teleoperation through direct PD control over the Internet. (We hide the slave side, which is symmetric to the master side.)

communication (with arbitrary packet loss, varying delay, data duplication/swapping, etc). To our knowledge, this simultaneous enforcement of the closed-loop passivity and the position/force coordination for the Internet teleoperation has never been achieved so far by other results (except for PSPM - see below). Also, being PD, our control framework proposed here is also easy and simple to implement.

Perhaps, the most closely-related work to our hybrid PD control is passive passive set-position modulation (PSPM) [Lee and Huang [2010]], which, also by exploiting the hybrid nature, can achieve the closed-loop passivity and the position/force coordination of the Internet teleoperation. Due to its selective activation of passifying action, this PSPM can also provide higher level of performance compared to other "time-invariant" schemes (e.g., PD control [Lee and Spong [2006]]; wave-method [Niemeyer and Slotine [2004]]). This PSPM, yet, requires more complicated implementation as compared to our hybrid PD control proposed here (although this complexity allows for the PSPM's flexibility: e.g., data processing). Our result here also sheds a new light on the fundamental limitation of the widely-used sampled-data PD master-slave coupling, given the robot's device damping, local sampling rate and the communication imperfectness.

The rest of this paper is organized as follows. System modeling and the formulation of sampled-data PD-control over the Internet are given in Sec. 2. The main result - passivity and position/force coordination - is proved in Sec. 3. Concluding remarks and comments on future research are given in Sec. 4.

2. PRELIMINARY

2.1 Modeling and Control Objectives

Consider the bilateral teleoperator shown in Fig. 2 where the master and slave robots' dynamics are given by n-degree-of-freedom (DOF) robotic systems, s.t. for $i = 1, 2$

$$M_i(x_i)\ddot{x}_i(t) + C_i(x_i, \dot{x}_i)\dot{x}_i(t) = u_i(t) + f_i(t) \qquad (1)$$

where $\star_1, \star_2$ refer to the values of master and slave sides respectively; $x_i \in \Re^n$ is the configuration; $u_i(t) \in \Re^n$ is the master-slave coupling control; $f_i(t) \in \Re^n$ represents human/environmental force; $M_i \in \Re^{n\times n}$ is symmetric positive-definite inertia matrix with its minimum eigenvalue strictly larger than zero; and $C_i \in \Re^{n\times n}$ is the Coriolis matrix. The well-known passivity property of system (1) is that $\dot{M}_i - 2C_i$ is skew-symmetric [Spong et al. [2006]].

For the teleoperator (1), we would like to design the controls $u_i(t)$ to achieve the following control objectives:

- *closed-loop passivity:* there exists a bounded constant $c \in \Re$ s.t.

$$\int_0^{\bar{t}} \left[f_1^T(\tau)\dot{x}_1(\tau) + f_2^T(\tau)\dot{x}_2(t)\right]d\tau \geq -c^2 \quad \forall \bar{t} \geq 0 \qquad (2)$$

- *position coordination:* if $f_i(t) = 0$

$$x_1(t) - x_2(t) \to 0 \qquad (3)$$

- *force reflection:* with $(\ddot{x}_i, \dot{x}_i) \to 0$

$$f_1(t) + f_2(t) \to 0. \qquad (4)$$

Following [Lee and Li [2003], Lee and Spong [2006]], we also define *controller passivity* as: $\exists d \in \Re$ s.t.

$$\int_0^{\bar{t}} \left[u_1^T(\tau)\dot{x}_1(\tau) + u_2^T(\tau)\dot{x}_2(\tau)\right]d\tau \leq d^2 \qquad \forall \bar{t} \geq 0 \qquad (5)$$

where d is a bounded constant. From Fig. 2, it is clear that if the master-slave coupling control over the Internet communication channels is passive, the closed-loop system is passive because both the master and the slave robots are passive. The following lemma summarizes this idea.

Lemma 1. (Lee and Li [2005]). Controller passivity (5) implies closed-loop passivity (2).

2.2 Sampled-Data PD-Control over the Internet

As mentioned in Sec. 1, the bilateral teleoperator in Fig. 2 is a hybrid system (i.e. continuous-time master and slave robots; sampled-data coupling control; discrete-time Internet communication channels). In order to incorporate this hybrid nature, following [Lee and Spong [2006], Huang and Lee [2010]], we design the sampled-data master-slave coupling control law $u_i(t), i = 1, 2$ s.t. for $t \in [t_k, t_{k+1})$

$$u_i(t) = \underbrace{-D\left[v_i(k) - \delta_i^k v_j(k - N_j^k)\right]}_{\text{delayed D-action}} \underbrace{-K\left[x_i(k) - x_j(k - N_j^k)\right]}_{\text{delayed P-action}} - \underbrace{B_i\dot{x}_i(t)}_{\text{device damping}} \qquad (6)$$

$$v_i(k) := \frac{x_i(k) - x_i(k-1)}{T} \qquad (7)$$

where t_k is the time of the k^{th} update instant, and $T = t_{k+1} - t_k > 0$ is the constant update interval; $(i, j) \in \{(1, 2), (2, 1)\}$; $v_i(k)$ is the numerical approximation (backward differentiation) of the velocity based on the position measurement; $D, K, B_i \in \Re^{n\times n}$ are the symmetric and positive-definite control gains and inherent device viscous damping; and $N_j^k \geq 0$ is the varying discrete index delay.

As shown in Fig. 3, the indexing delay N_j^k can capture such imperfectness of Internet communication as varying-delay, data swap, packet duplication. For the case of packet loss, this N_j^k is not well-defined since $x_j(k - N_j^k)$ and $v_j(k - N_j^k)$ are not received; and it is also desirable, for (6), to hold the previous set-position data x_j instead of suddenly change it to zero. Thus, if no packet arrives at t_k, we set N_j^k s.t., $N_j^k \Leftarrow N_j^{k-1} + 1$ to (artificially) duplicate the previous packet (see Fig. 3). With this definition of N_j^k, we assume there exists an upper bound $\bar{N}_j$, s.t. $\forall k \geq 0, \bar{N}_j \geq N_j^k$.

In contrast, duplication of the set-velocity data v_j in (6), yet, can violate passivity (see Appendix A). To prevent this, we utilize the duplication avoidance function δ_i^k in (6) defined s.t., for $(i, j) \in \{(1, 2), (2, 1)\}$

$$\delta_i^k = \begin{cases} 0 & \text{if } v_j(k - N_i^k) \text{ is duplicated} \\ 1 & \text{otherwise.} \end{cases} \qquad (8)$$

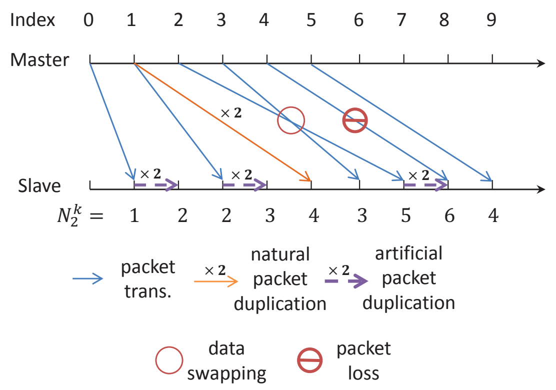

Fig. 3. Types of the communication defects that can be captured by indexing delay N_i^k.

where the first duplication case includes both the "real" packet duplication (from communication) and the "artificial" duplication (due to packet loss). This δ_i^k can be achieved simply by using packet numbering.

3. PASSIVITY AND PERFORMANCE OF HYBRID PD-BASED INTERNET TELEOPERATION

It is well-known that, with the imperfect Internet communication, the sampled-data PD control (6) can destabilize the teleoperator, particularly, with strong control gains. In the following theorem, we present the condition, under which the PD control (6) can enforce the closed-loop teleoperator passivity and the position/force coordination even in the presence of the discrete Internet's communication imperfectness.

Theorem 2. Consider the hybrid bilateral teleoperator (1) under the sampled-data PD control (6) over the imperfect Internet communication. Suppose the following condition is met,

$$B_i \succeq 2D + \Big[\frac{\bar{N}_1 + \bar{N}_2}{2} + 1\Big]TK \tag{9}$$

where $A \succeq B$ means $(A - B)$ is positive semi-definite. Also suppose $x_i(k) = x_i(0), \forall k < 0$. Then,

- the closed-loop passivity (2) is achieved;
- if $f_i(t) \equiv 0, \forall t \geq 0$ and
$$B_i \succ 2D + \Big[\frac{\bar{N}_1 + \bar{N}_2}{2} + 1\Big]TK \tag{10}$$
the master-slave position coordination is achieved s.t.
$$x_1(t) \to x_2(t) \tag{11}$$
where $A \succ B$ means that $(A - B)$ is positive-definite;
- if $\big(\dot{x}_1(t), \dot{x}_2(t), \ddot{x}_1(t), \ddot{x}_2(t)\big) \to 0$,
$$f_1(t) \to -f_2(t) \to K\big[x_1(t) - x_2(t)\big]. \tag{12}$$

Proof. 1) Let us denote the energy produced by both of the master and slave controllers by:

$$\begin{aligned}\epsilon(k) &= \int_{t_k}^{t_{k+1}} \big[u_1^T(\tau)\dot{x}_1(\tau) + u_2^T(\tau)\dot{x}_2(\tau)\big]d\tau \\ &= -\sum_{(i,j)\in\mathcal{E}} \Big\{\big[x_i(k) - x_j(k - N_j^k)\big]^T K\Delta x_i(k) \\ &\qquad + \big[v_i(k) - \delta_i^k v_j(k - N_j^k)\big]^T D\Delta x_i(k) \\ &\qquad + \int_{t_k}^{t_{k+1}} \dot{x}_i^T(\tau)B_i\dot{x}_i(\tau)d\tau\Big\} \end{aligned} \tag{13}$$

where $\Delta x_i(k) := x_i(k+1) - x_i(k), \mathcal{E} := \{(1,2),(2,1)\}$, which consists of the energy generated by P-action and D-action and B_i damping dissipation. According to Lemma 1, we only need to show the controller passivity. The key idea of the proof is that, with the passivity condition (9), the device damping B_i dissipates the undesired energy generated by the delayed P and D control actions.

Device Damping Dissipation

We denote the device damping dissipation in (13) by,

$$\begin{aligned}\epsilon_b(k) &:= -\int_{t_k}^{t_{k+1}} \big[\dot{x}_1^T(\tau)B_1\dot{x}_1(\tau) + \dot{x}_2^T(\tau)B_2\dot{x}_2(\tau)\big]d\tau \\ &\leq -\frac{1}{T}\big[\Delta x_1^T(k)B_1\Delta x_1(k) + \Delta x_2^T(k)B_2\Delta x_2(k)\big] \end{aligned} \tag{14}$$

where the inequality is due to the following inequality [Lee [2009], Lee and Huang [2010]]:

$$\int_{t_k}^{t_{k+1}} \dot{x}^T(\tau)B\dot{x}(\tau) \geq \frac{1}{T}\Delta x^T(k)B\Delta x(k). \tag{15}$$

Energy Generated by P-Action

We extract the energy generated by P-action in (13) and denote it as,

$$\begin{aligned}\epsilon_p(k) :=& -\big[x_1(k) - x_2(k - N_2^k)\big]^T K\Delta x_1(k) \\ & -\big[x_2(k) - x_1(k - N_1^k)\big]^T K\Delta x_2(k) \\ \leq& -\big[\hat{x}_1(k) - \hat{x}_2(k)\big]^T K\big[\Delta x_1(k) - \Delta x_2(k)\big] \\ & + \Delta x_1^T(k)K\Delta x_1(k) + \Delta x_2^T(k)K\Delta x_2(k) \\ & -\big[x_2(k) - x_2(k - N_2^k)\big]^T K\Delta x_1(k) \\ & -\big[x_1(k) - x_1(k - N_1^k)\big]^T K\Delta x_2(k) \end{aligned} \tag{16}$$

where $\hat{x}_i(k) := [x_i(k+1) + x_i(k)]/2$. For the derivation of the above inequality, please refer to Appendix A. The last two terms in (16) are the energy generated by the communication defects. We first focus on finding the upper bound of these two terms over the time.

By inserting intermediate terms $\sum_{j=k-N_1^k+1}^{k-1}[x_1(j) - x_1(j)]$, we have (for the master side),

$$\begin{aligned}\varepsilon_p^m &:= -\sum_{k=0}^{\bar{M}} \big[x_1(k) - x_1(k - N_1^k)\big]^T K\Delta x_2(k) \\ &= -\sum_{k=0}^{\bar{M}} \sum_{j=k-N_1^k}^{k-1} \Delta x_1^T(j)K\Delta x_2(k) \\ &\leq \sum_{k=0}^{\bar{M}} \sum_{j=k-N_1^k}^{k-1} \frac{1}{2}\big[\Delta x_1^T(j)K\Delta x_1(j) + \Delta x_2^T(k)K\Delta x_2^T(k)\big] \\ &\leq \sum_{k=0}^{\bar{M}} \sum_{j=k-\bar{N}_1}^{k-1} \frac{1}{2}\big[\Delta x_1^T(j)K\Delta x_1(j) + \Delta x_2^T(k)K\Delta x_2^T(k)\big] \\ &= \frac{1}{2}\sum_{k=0}^{\bar{M}} \sum_{j=k-\bar{N}_1}^{k-1} \Delta x_1^T(j)K\Delta x_1(j) \\ &\qquad + \frac{\bar{N}_1}{2}\sum_{k=0}^{\bar{M}} \Delta x_2^T(k)K\Delta x_2^T(k) \end{aligned} \tag{17}$$

By symmetry, we can derive a similar result for the slave side, s.t.

$$\begin{aligned}\varepsilon_p^s :=& -\sum_{k=0}^{\bar M}\left[x_1(k)-x_1(k-N_1^k)\right]^T K\Delta x_2(k)\\ \le& \frac{1}{2}\sum_{k=0}^{\bar M}\sum_{j=k-\bar N_2}^{k-1}\Delta x_2^T(j)K\Delta x_2(j)\\ &+\frac{\bar N_2}{2}\sum_{k=0}^{\bar M}\Delta x_1^T(k)K\Delta x_1(k). \end{aligned}\tag{18}$$

Then, combing (17) and (18), we have

$$\begin{aligned}&\varepsilon_p^m+\varepsilon_p^s\\ &\le \frac{1}{2}\sum_{k=0}^{\bar M}\sum_{j=k-\bar N_1}^{k-1}\Delta x_1^T(j)K\Delta x_1(j)+\frac{\bar N_2}{2}\sum_{k=0}^{\bar M}\Delta x_1^T(k)K\Delta x_1^T(k)\\ &+\frac{1}{2}\sum_{k=0}^{\bar M}\sum_{j=k-\bar N_2}^{k-1}\Delta x_2^T(j)K\Delta x_2(j)+\frac{\bar N_1}{2}\sum_{k=0}^{\bar M}\Delta x_2^T(k)K\Delta x_2^T(k).\end{aligned}\tag{19}$$

The following algebraic fact

$$\sum_{k=0}^{\bar M}\sum_{j=k-\bar N_i}^{k-1}\alpha_i(j)=\sum_{k=0}^{\bar M}\bar N_i\alpha_i(k)-\sum_{r=1}^{\bar N_i}r\alpha_i(r+\bar M-\bar N_i)\tag{20}$$

where $\alpha_i(k) := \Delta x_i^T(k)K\Delta_i(k)$, which is obtained by collecting $\alpha_i(k)$ and the residual terms of the left-hand side, can be used to rewrite (19) such as

$$\begin{aligned}&\varepsilon_p^m+\varepsilon_p^s\\ &\le\frac{\bar N_1+\bar N_2}{2}\sum_{k=0}^{\bar M}\left[\Delta x_1^T(k)K\Delta x_1(k)+\Delta x_2^T(k)K\Delta x_2(k)\right]\\ &\quad-\frac{1}{2}\sum_{r=1}^{\bar N_1}r\Delta x_1^T(r+\bar M-\bar N_1)K\Delta x_1(r+\bar M-\bar N_1)\\ &\quad-\frac{1}{2}\sum_{r=1}^{\bar N_2}r\Delta x_2^T(r+\bar M-\bar N_2)K\Delta x_2(r+\bar M-\bar N_2)\\ &\le\frac{\bar N_1+\bar N_2}{2}\sum_{k=0}^{\bar M}\left[\Delta x_1^T(k)K\Delta x_1(k)+\Delta x_2^T(k)K\Delta x_2(k)\right]\end{aligned}\tag{21}$$

where the first two terms are from the second, fourth terms of (19) and the first term of (20), and others come from the second term of (20). On the other hand, let us define the virtual spring energy as,

$$\phi(t)=\frac{1}{2}||x_1(t)-x_2(t)||_K^2$$

where $||\star||_K^2 := \star^T K\star$. It is easy to show that

$$\sum_{k=0}^{\bar M}\left[\hat x_1(k)-\hat x_2(k)\right]^T K\left[\Delta x_1(k)-\Delta x_2(k)\right]=\phi(\bar M+1)-\phi(0).$$

Hence, combining (16) and (21) yields,

$$\begin{aligned}\sum_{k=0}^{\bar M}\epsilon_p(k)\le&\Big(\frac{\bar N_1+\bar N_2}{2}+1\Big)\sum_{k=0}^{\bar M}\left[||\Delta x_1(k)||_K^2+||\Delta x_2(k)||_K^2\right]\\ &+\phi(0)-\phi(\bar M+1)\end{aligned}\tag{22}$$

Energy Generated by D-Action
Let us denote the energy generated by D-action as,

$$\begin{aligned}\epsilon_d(k)=&-\left[v_1(k)-\delta_1^k v_2(k-N_2^k)\right]^T D\Delta x_1(k)\\ &-\left[v_2(k)-\delta_2^k v_1(k-N_1^k)\right]^T D\Delta x_2(k)\\ =&-\frac{1}{T}\Big[\Delta x_1^T(k-1)D\Delta x_1(k)+\Delta x_2^T(k-1)D\Delta x_2(k)\\ &-\delta_1^k\Delta x_2^T(k-N_2^k-1)D\Delta x_1(k)\\ &-\delta_2^k\Delta x_1^T(k-N_1^k-1)D\Delta x_2(k)\Big]\end{aligned}\tag{23}$$

where the first line of (23) represents the energy generated by sampling effect, and the second & third lines are energy production caused by communication unreliability. Summing (23) up over the time, we have

$$\begin{aligned}\sum_{k=0}^{\bar M}\epsilon_d(k)\le&\frac{3}{2T}\sum_{k=0}^{\bar M}\left(||\Delta x_1(k)||_D^2+||\Delta x_2(k)||_D^2\right)\\ &+\frac{1}{2T}\sum_{k=0}^{\bar M}\left(\delta_1^k||\Delta x_2(k-N_2^k-1)||_D^2\right.\\ &\left.+\delta_2^k||\Delta x_1(k-N_1^k-1)||_D^2\right)\\ \le&\frac{2}{T}\sum_{k=0}^{\bar M}\left(||\Delta x_1(k)||_D^2+||\Delta x_2(k)||_D^2\right)\end{aligned}\tag{24}$$

where the second inequality is due to the following fact:

$$\sum_{k=0}^{\bar M}\delta_i^k||\Delta x_j(k-N_j^k-1)||_D^2\le\sum_{k=0}^{\bar M}||\Delta x_j(k)||_D^2\tag{25}$$

with $\delta_i^k = 0$ or 1. Note that this fact (25) does not hold without the duplication avoidance function δ_i^k (8).

Controller Passivity Validation
Combining (14), (22), (24), and passivity condition (9) we have

$$\begin{aligned}&\int_0^{\bar t}\left[u_1^T(\tau)\dot x_1(\tau)+u_2^T(\tau)\dot x_2(\tau)\right]d\tau=\sum_{k=0}^{\bar M}\epsilon(k)\\ &=\sum_{k=0}^{\bar M}\epsilon_d(k)+\epsilon_p(k)+\epsilon_b(k)\\ &\le\sum_{k=0}^{\bar M}\sum_{i=1}^{2}\Big[-\frac{1}{T}||\Delta x_i(k)||_{B_i}^2+\big(\frac{\bar N_1+\bar N_2}{2}+1\big)||\Delta x_i(k)||_K^2\\ &+\frac{2}{T}||\Delta x_i(k)||_D^2\Big]+\phi(0)-\phi(\bar M+1)\le\phi(0)=:d^2\end{aligned}\tag{26}$$

which implies the controller passivity and hence the closed-loop passivity following Lemma 1.

2) Our position coordination proof here roughly follows the similar proof in [Lee and Huang [2010]]. The first step is to show $\dot x_i(t)\to 0$. Let us define

$$\begin{aligned}\tilde u_i(t):=&-D\left[v_i(k)-\delta_i^k v_j(k-N_j^k)\right]\\ &-K\left[x_i(k)-x_j(k-N_j^k)\right]-\tilde B_i\dot x_i(t)\end{aligned}\tag{27}$$

where $\tilde B_i = 2D + \left[\frac{\bar N_1+\bar N_2}{2}+1\right]TK$. The kinetic energy of the system can be written as,

$$\kappa(t):=\frac{1}{2}\dot x_1^T M_1(x_1)\dot x_1(t)+\frac{1}{2}\dot x_2^T M_2(x_2)\dot x_2(t).\tag{28}$$

Using (1), $f_i(t)\equiv 0$, $\dot M_i - 2C_i$ being skew-symmetric and condition (10), we can find small but strictly positive definite $B_{\varepsilon i}\in\Re^{n\times n}$ s.t.

$$\begin{aligned}\frac{d}{dt}\kappa(t) =& u_1^T(t)\dot{x}_1(t) + u_2^T(t)\dot{x}_2(t) \\ =& \tilde{u}_1^T(t)\dot{x}_1(t) + \tilde{u}_2^T(t)\dot{x}_2(t) \\ &- \dot{x}_1^T(t)B_{\varepsilon 1}\dot{x}_1(t) - \dot{x}_2^T(t)B_{\varepsilon 2}\dot{x}_2(t). \end{aligned} \tag{29}$$

Integrating (29) over the time with controller passivity (5),

$$\kappa(t) \leq d^2 + \kappa(0) - \int_0^t \left[\dot{x}_1^T(t)B_{\varepsilon 1}\dot{x}_1(\tau) + \dot{x}_2^T(\tau)B_{\varepsilon 2}\dot{x}_2(\tau)\right]d\tau$$

which indicates $\dot{x}_i$ is bounded (with the assumption that the minimum eigenvalue of M_i is strictly larger than 0) and we further have

$$\int_0^t \left[\dot{x}_1^T(t)B_{\varepsilon 1}\dot{x}_1(\tau) + \dot{x}_2^T(\tau)B_{\varepsilon 2}\dot{x}_2(\tau)\right]d\tau \leq d^2 + \kappa(0). \tag{30}$$

Since $d^2, \kappa(0)$ are bounded constant, and $B_{\varepsilon i}$ is strictly positive-definite, (30) means $\dot{x}_i$ is square-integrable. From the closed-loop dynamics (with $f_i(t) \equiv 0$)

$$\begin{aligned} &M_i(x_i)\ddot{x}_i + C_i(x_i, \dot{x}_i)\dot{x}_i(t) + B_i\dot{x}_i(t) = \\ &- D\big[v_i(k) - \delta_i^k v_j(k - N_j^k)\big] - K\big[x_i(k) - x_j(k - N_j^k)\big] \end{aligned} \tag{31}$$

with the boundedness of $\dot{x}_i, v_i(k)$, and that $C(x_i, \dot{x}_i)$ is linear w.r.t. bounded $\dot{x}_i, \partial M_i/\partial x_i$, we can then conclude that $\ddot{x}_i$ is uniformly bounded. By Barbalat's lemma [Spong et al. [2006]], $\dot{x}_i \to 0$ as $t \to \infty$. Hence, it is clear that when t is large (31) becomes: for $t \in T_k := [t_k, t_{k+1})$

$$M_i(x_i)\ddot{x}_i(t) \to -K\big[x_i(k) - x_j(k)\big] \tag{32}$$

The next step is to show $x_i(t) \to x_j(t)$. The proof is done by contradiction with this idea: following (32), assume $x_i(k) \nrightarrow x_j(k)$, the nonzero $\ddot{x}_i(t)$ will drive $\dot{x}_i(t)$ away from 0 which is contradict to $\dot{x}_i(t) \to 0$ (the detailed proof can be found in Appendix B). Hence, we conclude $x_i(t) \to x_j(t)$ which further implies $\ddot{x}_i(t) \to 0$ from (32).

3) with $\dot{x}_i(t) \to 0, \ddot{x}_i(t) \to 0$, (1) can be rewritten as,

$$f_i(t) \to K\big[x_i(k) - x_j(k - N_j^k)\big] \to K\big[x_i(k) - x_j(k)\big]$$

with the assumption that the indexing delay is upper bounded. ■

Thus, by setting the control gains according to (9), the sampled-data PD control (6) ensures the closed-loop passivity and also the interaction stability with passive human/environments. Note also that, with the explicit position feedback (i.e. P-action), the PD control (6) can guarantee steady-state position coordination under (10) even in the presence of packet loss.

Our PD control (6) is simple and easy to implement: with its gains satisfying (6), we can simply use the most current position data for the P-action when no data is received (i.e. blank packet), regardless of whether there exist data duplication or not; while, for the D-action, we need to use the duplication avoidance function, which can be easily implemented by using the packet numbering.

The passivity condition (9) requires the device damping B_i be large enough to passify the unwanted energy produced by communication imperfectness (requires $(\bar{N}_1 + \bar{N}_2)KT/2$) and sampling effects (requires $2D + KT$). Note from (9) that, if $\bar{N}_iT$ (i.e. maximum data separation with all the Internet's imperfectness taken into account) is small, B_i may handle with a large K, although D adversely affects this (passifying) B_i regardless of $\bar{N}_iT$. This condition (9) in fact reveals the **fundamental (passivity/stability) limitation** of the sampled-data PD control (6), which is very widely-used in practice, given the device viscous damping B_i, communication imperfectness (i.e. $\bar{N}_i$), and robot's local sampling rates T.

For the sampled-data PD control (6), T is the robot's local sampling rate. However, we may still use the condition (9) for the case where the local sampling rate is very fast (this is true in many teleoperation systems), the Internet data reception rate much slower than that, and the updates of the P and D actions of (6) are triggered by the Internet packet reception. In this case, T in (6) will be the Internet data reception rate; and B_i the control damping, with $B_i\dot{x}_i$ in (6) being the damping control, which can be thought of as a continuous-time signal with the fast local sampling rate.

Although our results are derived for sampled-data PD control and discrete-time communication links, it has close relation with the results derived for the continuous-time PD control and communication channels [Lee and Spong [2006], Nuno et al. [2008]]. Let us make the sampling and communication update rate T converge to 0. Then, the teleoperation system becomes continuous and $\bar{N}_iT$ equals to the maximum transmission delay τ_i (we adopt the notation in [Lee and Spong [2006]]). Moreover, the damping requirement $2D + KT$, which is due to the sampling effects, vanishes in the continuous-time case. Assuming (9) holds for infinitesimal T, it then becomes the same condition given in [Lee and Spong [2006], Nuno et al. [2008]], which implies our result is a discrete-time extension to their works. However, when $T \to 0$ with $\bar{N}_iT \to \tau_i$, the indexing delay $\bar{N}_i$ goes to ∞ which violates our assumption that $\bar{N}_i$ is upper bounded. Filling this gap requires different techniques and will be reported in our future work.

The PD control (6) is conservative with its performance degrading rapidly when $\bar{N}_iT$ becomes large. This is similar to other "time-invariant" techniques (e.g., PD control [Lee and Spong [2006]]; wave-method [Niemeyer and Slotine [2004]]), which applies (conservative) passifying action (i.e. (9)) designed for the "worst" case, that is not so often occurring during typical operations. This conservatism may be substantially reduced by using (time-variant) PSPM [Lee and Huang [2010]], which only selectively activates the passifying action when necessary.

4. CONCLUSIONS

In this paper, we propose a novel hybrid passivity-enforcing PD-based control framework for the bilateral teleoperation over imperfect Internet communication with arbitrary varying-delay, packet-loss, data duplication/swapping, etc. Our result also reveals the fundamental limitation on the widely-used sampled-data PD teleoperation control, given the device damping, communication imperfectness and local servo-rates.

Some directions we will pursue for future works include: 1) relax the passivity condition (9), particularly, ease the limitation on the un-adjustable device damping B_i [1]; 2) extend the results to continuous-time PD control and communication (with varying-delay and temporary

[1] A new virtual-proxy based control framework, which utilizes the discrete damping rather than the device damping to passify the imperfect Internet, will be reported in [Huang and Lee [2011]].

connection blackout); and 3) experimental validations of the proposed results.

Appendix A. PROOF OF INEQUALITY (16)

$$\begin{aligned}\epsilon_p(k) =& -\left[x_1(k) - x_2(k - N_2^k)\right]^T K\Delta x_1(k)\\ & -\left[x_2(k) - x_1(k - N_1^k)\right]^T K\Delta x_2(k)\\ =& -\left[x_2(k) - x_2(k - N_2^k)\right]^T K\Delta x_1(k)\\ & -\left[x_1(k) - x_1(k - N_1^k)\right]^T K\Delta x_2(k)\\ & -\left[x_1(k) - x_2(k)\right]^T K\Delta x_1(k)\\ & -\left[x_2(k) - x_1(k)\right]^T K\Delta x_2(k) \qquad \text{(A.1)}\end{aligned}$$

The last two lines can be further written as

$$\begin{aligned}&-\left[x_1(k) - x_2(k)\right]^T K\left[\Delta x_1(k) - \Delta x_2(k)\right]\\ &\quad = -\left[\hat{x}_1(k) - \hat{x}_2(k)\right]^T K\left[\Delta x_1(k) - \Delta x_2(k)\right]\\ &\quad + \frac{1}{2}\left[\Delta x_1(k) - \Delta x_2(k)\right]^T K\left[\Delta x_1(k) - \Delta x_2(k)\right] \quad \text{(A.2)}\end{aligned}$$

where the last line is upper bounded by

$$\Delta x_1^T(k) K\Delta x_1(k) + \Delta x_2^T(k) K\Delta x_2(k) \qquad \text{(A.3)}$$

Substituting (A.2) and (A.3) into (A.1) yields (16).

Appendix B. COMPLEMENTARY PROOF OF POSITION COORDINATION

To be rigorous, let us start with (31) and $\lim_{t\to\infty} \dot{x}_i(t) = 0$, which have been justified by previous proof. Assume $x_i(k)$ does not converge to $x_j(k - N_j^k)$, i.e. for some $\epsilon > 0$, $\forall N_a \geq 0$, there exists $k_\epsilon \geq N_a$ s.t.

$$\left\| K[x_i(k_\epsilon) - x_j(k_\epsilon - N_j^{k_\epsilon})]\right\| > \epsilon \qquad \text{(B.1)}$$

Also, by $\dot{x}_i(t) \to 0$ and boundedness of C_i, B_i, D, there exists $N_b \geq 0$ s.t. $\forall k \geq N_b$ and $t \in T_k := [t_k, t_{k+1})$

$$\left\|C_i(x_i, \dot{x}_i)\dot{x}_i(t) + B_i\dot{x}_i(t) + D\left[v_i(k) - \delta_i^k v_j(k - N_j^k)\right]\right\| \leq \frac{\epsilon}{2}. \qquad \text{(B.2)}$$

Hence, by choosing $N_a \geq N_b$ there exists $k_\epsilon \geq N_a$ which makes (B.1) and (B.2) hold at the same time. Let us denote $\rho(k_\epsilon) := -K[x_i(k_\epsilon) - x_j(k_\epsilon - N_j^{k_\epsilon})]$. Then, by (31) and (B.1)-(B.2), it is clear that $\forall t \in T_{k_\epsilon}$

$$M_i(x_i)\ddot{x}_i(t) \in \left\{y \in \Re^n \middle| \|y - \rho(k_\epsilon)\| \leq \frac{\epsilon}{2}\right\}$$

Note that $\rho(k_\epsilon)$ is constant during $[t_{k\epsilon}, t_{k\epsilon+1})$ and is outside of the closed ball $B_\epsilon^c := \{y \in \Re^n | \|y\| \leq \epsilon\}$. It is clear that

$$\lambda_{\max}(M_i)\|\dot{x}_i(t_{k_\epsilon+1}) - \dot{x}_i(t_{k_\epsilon})\| \geq \left\|\int_{T_{k_\epsilon}} M_i(x_i)\ddot{x}_i(t)dt\right\| > \frac{\epsilon T}{2}$$

Note that k_ϵ can be arbitrarily large. This inequality then contradicts to $\lim_{t\to\infty} \dot{x}_i(t) = 0$. Hence, $x_i(k) \to x_j(k - N_j^k)$, which further implies $x_i(t) \to x_j(t)$ due to $\dot{x}_i(t), \dot{x}_j(t) \to 0$. ■

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