Abstract: We propose a novel control scheme for teleoperators consisting of a pair of multi-degree-of-freedom (DOF) nonlinear robotic systems under a constant communication time delay. By passifying the communication and control blocks together, the proposed control scheme guarantees energetic passivity of the closed-loop teleoperator in the presence of parametric uncertainty and a constant communication delay without relying on widely utilized scattering or wave formalisms. The proposed control scheme also achieves master-slave position coordination and bilateral static force reflection. The proposed control scheme is symmetric in the sense that the control and communication laws of the master and slave are of the same form. Simulations are performed to validate properties of the proposed control scheme. Copyright © 2005 IFAC

Keywords: nonlinear teleoperator, time-delay, passivity, Lyapunov-Krasovskii functionals, Parseval’s identity

1. INTRODUCTION

Energetically, a closed-loop teleoperator is a two-port system (see figure 1). Thus, the foremost goal of the control (and communication) design is to ensure interaction safety and coupled stability (Colgate, 1994) when it is mechanically coupled with a broad class of slave environments and human operators. For this, energetic passivity (i.e. mechanical power as the supply rate (Willems, 1972)) of the closed-loop teleoperator has been widely utilized as the control objective (Anderson and Spong, 1989; Niemeyer and Slotine, 1991; Lawrence, 1993; Stramigioli et al., 2002; Lee and Li, 2003a; Lee and Li, 2002). This is because 1) the feedback interconnection of the passive teleoperator with any passive environments/humans is necessarily stable (Colgate, 1994); and 2) in many cases, slave environments are passive (e.g. pushing a wall) and humans can be assumed as passive systems (Hogan, 1989). Also, passive teleoperator would be potentially safer to interact with, since the maximum extractable energy from it is bounded, thus, possible damages on environments/humans are also limited.

How to ensure passivity of the time-delayed bilateral teleoperation was a long standing problem. In (Anderson and Spong, 1989), scattering theory was proposed to passify the delayed-communication, and passivity of the teleoperator is ensured for arbitrary constant time-delay. In (Niemeyer and Slotine, 1991), this result is extended and the notion of wave variables was introduced. Since these two seminal works, scattering
2. PROBLEM FORMULATION

2.1 Plant

Let us consider a nonlinear mechanical teleoperator consisting of a pair of \( n \)-DOF robotic systems:

\[
\begin{align*}
M_1(q_i)\dot{q}_1(t) + C_1(q_i, \dot{q}_1)\dot{q}_1 &= T_1(t) + F_1(t), \\
M_2(q_2)\dot{q}_2(t) + C_2(q_2, \dot{q}_2)\dot{q}_2 &= T_2(t) + F_2(t),
\end{align*}
\]

where \( q_i, F_i, T_i \in \mathbb{R}^n \) are configurations, human/environmental force, and controls, and \( M_i(q_i), C_i(q_i, \dot{q}_i) \in \mathbb{R}^{n \times n} \) are symmetric and positive-definite inertia matrices and Coriolis matrices s.t. \( M_i(q_i) \ 2C_i(q_i, \dot{q}_i) \) are skew-symmetric (\( i = 1, 2 \)).

Considering a constant time-delay \( 0 \) (see figure 1), we will define the controls \( T_1(t), T_2(t) \) in (1)-(2) to be functions of the current and stored local information and the remote information delayed by the constant time-delay \( 0 \) s.t. \( T_i(t) := T_i(\hat{q}_i(t), \dot{\hat{q}}_i(t), q_i(t - \tau), \dot{q}_i(t - \tau), \dot{q}_j(t), \dot{q}_j(t - \tau)) \in \mathbb{R}^n \), where \( i, j \in \{1, 2\}, i \neq j \).

2.2 Control Objectives

We would like to design the controls \( T_1(t), T_2(t) \) to achieve the following master-slave position coordinate: when \( (F_1(t), F_2(t)) = 0 \),

\[
q_E(t) := q_1(t) \quad q_2(t) \to 0, \quad t \to \infty.
\]

Fig. 1. Closed-loop teleoperator as a two-port system.

We would also like to achieve the bilateral force reflection: when \( (\dot{\hat{q}}_1(t), \dot{\hat{q}}_2(t), \hat{q}_1(t), \hat{q}_2(t)) \to 0 \),

\[
F_1(t) \to F_2(t).
\]

For safe interaction and coupled stability, we would like to enforce the following energetic passivity condition for the two-port teleoperator (1)-(2): there exists a finite constant \( d \in \mathbb{R} \) s.t. \( \forall t \),

\[
\int_0^t F_1^T(\dot{q}_1) + F_2^T(\dot{q}_2) \, d \quad d^2,
\]

e. m. maximum extractable energy from the two-port closed-loop teleoperator is always bounded.

Let us also define controller passivity condition (Lee, 2004; Lee and Li, 2003b) : there exists a finite constant \( c \in \mathbb{R} \) s.t. \( \forall t \),

\[
\int_0^t T_1^T(\dot{q}_1) + T_2^T(\dot{q}_2) \, d \quad \bullet \quad c^2,
\]

e. energy generated by the two-port controller (figure 1) is always limited.

**Lemma 1.** (Lee, 2004; Lee and Li, 2003b) For the mechanical teleoperator (1)-(2), controller passivity (6) implies energetic passivity (5).

**Proof:** Let us define the total kinetic energy

\[
\kappa_f(t) := \frac{1}{2}q_1^T M_1(q_1)\dot{q}_1 + \frac{1}{2}q_2^T M_2(q_2)\dot{q}_2,
\]

then, using the dynamics (1)-(2) and its skew-symmetric property, we have

\[
\frac{d}{dt}\kappa_f(t) = T_1^T(\dot{q}_1 + \dot{q}_2) + F_1^T(\dot{q}_1) + F_2^T(\dot{q}_2).
\]

Thus, by integrating above equality with the controller passivity condition (6) and the fact that \( \kappa_f(t) \to 0 \), we have, \( \forall t \),

\[
\int_0^t [F_1^T(\dot{q}_1) + F_2^T(\dot{q}_2)] \, d = \kappa_f(0) \cdot c^2 =: \Delta^2.
\]

3. CONTROL DESIGN

To achieve coordination (3), force reflection (4), and passivity (5), we design the master and slave controls \( T_1(t), T_2(t) \) to be

\[
T_1(t) := (K_d + 2K_v + P_d)\dot{q}_1(t) + K_p\dot{q}_1(t) + K_vq_1(t),
\]

\[
T_2(t) := (K_d + 2K_v + P_d)\dot{q}_2(t) + K_p\dot{q}_2(t) + K_vq_2(t).
\]
where \( K_v, K_p \in \mathbb{R}^{n \times n} \) are symmetric and positive-definite proportional-derivative (PD) control gains, \( K_d \in \mathbb{R}^{n \times n} \) is the the dissipation gain to enforce energetic passivity and designed to satisfy the following “gain-setting” condition s.t.

\[
K_d \frac{2\sin\omega w}{w} K_p \preceq 0, \quad \forall w \in \mathbb{R},
\]

(i.e. \( K_d \frac{2\sin\omega w}{w} K_p \) is positive-semidefinite), and \( P_{\ast} \in \mathbb{R}^{n \times n} \) is an additional dissipation gain (e.g. inherent device damping) to be exploited for the position coordination proof in theorem 2. Since \( \frac{2\sin\omega w}{w} 0, \forall w \in \mathbb{R} \), one possible solution for the condition (11) is given by

\[
K_d = 2K_p.
\]

**Theorem 2.** Consider the mechanical teleoperator (1)-(2) under the controls (9)-(10) designed to satisfy the gain-setting condition (11).

1) The closed-loop teleoperator is passive (i.e. satisfies (5)) regardless of parametric uncertainty in (1)-(2) (i.e. robust passivity (Lee and Li, 2003b));

2) Suppose that the human operator and slave environment are energetically passive, i.e. \( \exists \) finite constants \( d_1, d_2 \in \mathbb{R} \) s.t.

\[
\int_0^t \left[ E^T(q_s) \right] \left[ \begin{array}{c}
\dot{q}_s(t) \\
dq_s(t)
\end{array} \right] dt, (13)
\]

mechanical power inflow to human/environment

\( \forall t \) (\( * \in \{1, 2\} \)), i.e. the maximum extractable energy from them are bounded. Then, \( \dot{q}_1(t), \dot{q}_2(t) \in L_\infty \). Thus, if the human and slave environment are \( L_\infty \)-stable input-output impedance maps, \( F_1(t), F_2(t) \in L_\infty \);

3) Suppose that the human and slave environment are passive in the sense of (13). Suppose further that \( M_{ij}^k(q_i), M_{ijk}^{k_1}(q_i) \) and \( \dot{\alpha}_i, \dot{\alpha}_i^k, \dot{\alpha}_i^{k_1} \) are all bounded, where \( \dot{\alpha}_i, \dot{\alpha}_i^k, \) and \( \dot{\alpha}_i^{k_1} \) are the jk-th and the \( \alpha \)-th components of \( M_i(q_i) \) and \( \dot{q}_i \). Then, \( \dot{q}_i(t) = q_i(t) \) \( \dot{q}_2(t) \) is bounded \( \forall t \). Moreover, if \( (F_1(t), F_2(t)) = 0, (\dot{q}_1(t), \dot{q}_2(t)) \to 0 \) (i.e. (3) is achieved);

4) If \( (\dot{q}_1(t), \dot{q}_2(t), \dot{q}_1(t), \dot{q}_2(t)) \to 0 \), then, \( F_1(t) \to F_2(t) \to K_p(q_1(t) \dot{q}_2(t)) \) (i.e. (4) is achieved).

**Proof:** 1) Let us denote the mechanical power generated by the controls (9)-(10) by

\[
s_t(t) := T_1^T(t) \dot{q}_1(t) + T_2^T(t) \dot{q}_2(t) = s_v(t) + s_p(t) = P(t),
\]

where \( P(t) \) is the following quadratic form:

\[
P(t) := \begin{bmatrix}
q_1(t) \\
q_2(t)
\end{bmatrix}^T \begin{bmatrix}
P_{\ast} & 0 \\
0 & P_{\ast}
\end{bmatrix} \begin{bmatrix}
q_1(t) \\
q_2(t)
\end{bmatrix}, (14)
\]

and the terms \( s_v(t) \) and \( s_p(t) \) are defined by

\[
s_v(t) := 2\hat{q}_1^T K_v \dot{q}_1 + \hat{q}_1^T K_v \dot{q}_2 + 2\hat{q}_2^T K_v \dot{q}_2,
\]

\[
s_p(t) := 2\hat{q}_1^T K_d \dot{q}_1 + \hat{q}_1^T K_d \dot{q}_2 + 2\hat{q}_2^T K_d \dot{q}_1 + 2\hat{q}_2^T K_d \dot{q}_2,
\]

\[
(15, 16)
\]

where we denote \( (\ast(t), \ast(t)) \) by \( (\ast, \ast) \).

Similar to the Lyapunov-Krasovskii functionals (Gu and Niculescu, 2003), we define

\[
V_v(t) := \frac{1}{2} \int_0^t \dot{q}_1^T(t) K_v \dot{q}_1(t) + \dot{q}_2^T(t) K_v \dot{q}_2(t) dt, (17)
\]

where \( q_v(t) := q_1(t) + q_2(t) \in \mathbb{R}^n \), and its time-derivative is given by

\[
\frac{d}{dt} V_v(t) = \frac{1}{2} \dot{q}_1^T(t) K_v \dot{q}_1(t) + \dot{q}_2^T(t) K_v \dot{q}_2(t), (19)
\]

where we denote \( (\ast(t), \ast(t)) \) by \( (\ast, \ast) \). Thus, by integrating the inequality (19), we have

\[
\int_0^t s_v(d) \quad V_v(t) + V_v(0), (20)
\]

Let us consider the supply rate \( s_p(t) \) in (16), using the definition of \( q_v(t) = q_1(t) + q_2(t) \) in (3) and the following inequality similar to (18) s.t.

\[
2 \dot{q}_1^T K_v \dot{q}_1 + 2 \dot{q}_2^T K_v \dot{q}_2 \dot{q}_1^T K_d \dot{q}_2,
\]

\[
(21)
\]

we have

\[
s_p(t) \quad 2 \dot{q}_1^T K_v \dot{q}_1 + 2 \dot{q}_2^T K_v \dot{q}_2 \dot{q}_1^T K_d \dot{q}_2
\]

\[
= \frac{1}{2} \dot{q}_1^T(t) K_d \dot{q}_1(t) + \dot{q}_2^T(t) K_d \dot{q}_2(t)
\]

\[
+ \frac{\dot{q}_1^T(t) K_d \dot{q}_2(t)}{2} \frac{\dot{q}_2^T(t) K_d \dot{q}_2(t)}{2} \frac{d}{dt} V_p(t), (22)
\]

where \( V_p(t) \) is the spring potential energy, i.e.

\[
V_p(t) := \frac{1}{2} \dot{q}_2^T(t) K_v \dot{q}_2(t), (23)
\]

Let us define truncated signal \( \hat{q}_E(t) \) of \( q_v(t) \) s.t.

\[
\hat{q}_E(t) := \begin{bmatrix}
\dot{q}_v(t) & \dot{q}_v(t)
\end{bmatrix}
\quad \text{if} \quad t \in [0, t]
\]

\[
\text{otherwise}
\]

\[
(24)
\]

with its Fourier transform given by \( V_E(w) = \int_{-\infty}^{\infty} \hat{q}_E(t) e^{jwtd} dt = \int_0^t \hat{q}_E(t) e^{jwtd} dt \) (\( \omega = \sqrt{-1} \)).

Then, using and Parseval’s identity (Goldberg, 1961), the gain-setting condition (11), and the
fact that $\dot{V}_E^T(w) K_d \dot{V}_E^T(w)$ and $\dot{V}_E(w) K_p \dot{V}_E^T(w)$ are even-functions w.r.t. $w$, we have: $\forall t \neq 0,$
\[
\int_0^t s_p(\cdot) dt \quad V_p(t) + V_p(0)
\]
\[
\frac{1}{2} \int_{-\infty}^{\infty} \dot{V}_E^T(w) K_d \dot{V}_E^T(w) dw
\]
\[
+ \int_{-\infty}^{\infty} \dot{V}_E(w) K_p \frac{1}{jw} \dot{V}_E^T(w) dw
\]
\[
= V_p(t) + V_p(0)
\]
\[
\frac{1}{2} \int_{-\infty}^{\infty} \dot{V}_E^T(w) K_d \frac{2 \sin w}{w} K_p \dot{V}_E^T(w) dw
\]
\[
\bullet \quad V_p(t) + V_p(0), \tag{25}
\]
where we denote the complex conjugate transpose of a complex vector $\ast \in \mathbb{C}^n$ by $\ast$ (i.e. $\ast = \ast^T$).

Thus, by summing up (20) and (25) with the fact that $V_p(0)$ and $V_E(0) \neq 0$, we can prove controller passivity (6) s.t. for all $\forall t \neq 0$,
\[
\int_0^t T_1^T(\cdot) \hat{q}_1(\cdot) + T_2^T(\cdot) \hat{q}_2(\cdot) dt
\]
\[
= \int_0^t [s_q(\cdot) + s_p(\cdot)] \cdot \int_0^t P(\cdot) dt
\]
\[
\bullet \quad V_p(t) + V_p(0) = c^2
\]
\[
\tag{26}
\]
where $V_p(0)$ will be zero if $(\hat{q}_1(t), \hat{q}_2(t)) = 0 \forall t \in (-\infty, 0)$ and $V_p(0)$ is the initial spring potential energy. Thus, from lemma 1, energetic passivity (5) follows. Since controller passivity (6) doesn’t depend on the parameters in (1)-(2), this achieved passivity is robust against parametric uncertainty.

2) By integrating the equality (8) with the controller passivity (26) and the human/slave environment passivity (13), we have, for all $t \neq 0,$
\[
\kappa_f(t) + V_p(t) + V_p(0)
\]
\[
\bullet \quad \kappa_f(0) + V_p(0) + V_p(0) \int_0^t P(\cdot) dt + d_1^2 + d_2^2
\]
\[
\tag{27}
\]
where $P(t) \neq 0$. Therefore, $\kappa_f(t)$ is bounded, thus, $\hat{q}_1(t), \hat{q}_2(t)$ are also bounded $\forall t \neq 0$ (i.e. $\hat{q}_1(t), \hat{q}_2(t) \in L_\infty$). Moreover, if the human and slave environment are $L_\infty$-stable impedance maps, $F_1(t), F_2(t) \in L_\infty$.

3) Boundedness of $q_2(t) = q_1(t)$ $q_2(t)$ is a direct consequence of the inequality (27) and the definition of $V_p(t)$ in (23).

First step of the convergence proof is to show that $(\hat{q}_1(t), \hat{q}_2(t)) \to 0$. Suppose that $F_1(t), F_2(t) = 0$, $\forall t \neq 0$. Then, from (27) with $d_1 = d_2 = 0$ and the boundedness of $P$, $M_1(q_1), M_2(q_2)$, we have:
\[
\kappa_f(t) \quad \kappa_f(0) + c^2 \int_0^t P(\cdot) dt
\]
\[
\bullet \quad \kappa_f(0) + c^2 \int_0^t \kappa_f(\cdot) dt
\]
\[
\forall t \neq 0, \text{ where } P(t) \text{ and } c \text{ are defined in (14) and (26), and } c > 0 \text{ is a constant scalar. Here, since }
\]
\[
\kappa_f(t) \neq 0, \text{ the term } \int_0^{\infty} \kappa_f(\cdot) dt \text{ is monotonically increasing and upper bounded, thus, it converges to a limit. Therefore, following Barbalat’s lemma, if } \kappa_f(t) \text{ is uniformly continuous, } \kappa_f(t) \text{ will also converge to } 0 \text{ (i.e. } \hat{q}_1(t), \hat{q}_2(t) \to 0). \text{ To show this, let us consider } \frac{1}{t} \kappa_f(t). \text{ Then, from (8) with } F_1(t) = F_2(t) = 0, \text{ we have } \frac{1}{t} \kappa_f(t) = T_1^T(t) \hat{q}_1(t) + T_2^T(t) \hat{q}_2(t), \text{ where } \hat{q}_1(t), \hat{q}_2(t) \text{ are bounded from item 2 of this theorem), and } T_1(t), T_2(t) \text{ are also bounded from their definitions (9)-(10) with bounded } q_E(t) = q_1(t) \text{ } q_2(t) \text{ (from item 2 of this theorem). Thus, } \kappa_f(t) \text{ is uniformly continuous, therefore, } \kappa_f(t) \to 0 \text{ and } (\hat{q}_1(t), \hat{q}_2(t)) \to 0.$

Second step is now to show that $(\hat{q}_1(t), \hat{q}_2(t)) \to 0$ to estimate $q_1(t)$ and $q_2(t)$. Let us consider the teleoperator dynamics (1)-(2) with $F_1(t) = F_2(t) = 0$, where, as shown in the above paragraph, the controls $T_1(t), T_2(t)$ are bounded. Also, from the boundedness assumption of $\frac{\partial M_1}{\partial q_1}$, the Coriolis terms $C_1(q_1, q_2, \dot{q}_1, \dot{q}_2, i = 1, 2)$ in (1)-(2) are bounded. Thus, the accelerations $\ddot{q}_1(t), \ddot{q}_2(t)$ are also bounded $\forall t \neq 0$. Now, let us consider the acceleration $\ddot{q}_1(t)$ in (1)-(2) (with $F_1(t) = 0$):
\[
\ddot{q}_1 = M_1^{-1}(q_1) C_1(q_1, \dot{q}_1, \dot{q}_2, i = 1, 2) \text{ in (1)-(2) are bounded due to the boundedness of } \ddot{q}_1(t), \ddot{q}_2(t), \langle q_1(t), \ddot{q}_2(t) \rangle, \langle \frac{\partial M_1}{\partial q_1}(q_1) \frac{\partial M_1}{\partial q_2}(q_1) \rangle \text{ (from the boundedness assumption on } \frac{\partial M_1}{\partial q_1}(q_1), \frac{\partial M_1}{\partial q_2}(q_1)), \text{ and } \frac{\partial^2 M_1}{\partial q_1^2}(q_1) \text{ (from the boundedness assumption on } \frac{\partial M_1}{\partial q_1}(q_1), \frac{\partial M_1}{\partial q_1}(q_1)) \text{ and } \frac{\partial}{\partial q_2} C_1(q_1, \dot{q}_2). \text{ This implies that the right hand side of } (29) \text{ is uniformly continuous. Thus, } \hat{q}_1(t), \hat{q}_2(t) \text{ are also uniformly continuous. Therefore, following Barbalat’s lemma, } (\hat{q}_1(t), \hat{q}_2(t)) \to 0 \text{ as } (\dot{q}_1(t), \dot{q}_2(t)) \to 0. \text{ Moreover, from the dynamics (1)-(2) with } (\hat{q}_1(t), \hat{q}_2(t), \dot{q}_1(t), \dot{q}_2(t)) \to 0 \text{ and } F_1(t) = F_2(t) = 0, \text{ we have } K_p(q_1(t), q_2(t)), \text{ i.e. } q_1(t) \to q_2(t), \text{ since } K_p \text{ is positive-definite.}

4) Suppose that $(\hat{q}_1(t), \hat{q}_2(t), \dot{q}_1(t), \dot{q}_2(t)) \to 0$. Then, from the dynamics (1)-(2) with the controls (9)-(10), we have:
\[
F_1(t) \to K_p(q_1(t), q_2(t)),
\]
\[
F_2(t) \to K_p(q_2(t), q_1(t)),
\]
\[
\tag{30}
\]
where we use $\hat{q}_1(t) \to 0$ and $\dot{q}_1(t) \to q_1(t)$ as $(\hat{q}_1(t), \dot{q}_1(t), \dot{q}_2(t), \ddot{q}_2(t)) \to 0.$

In the human/environment passivity condition (13), the negative sign in the integration comes from the fact that the power in w.r.t. to those systems are given by $(F_1(t))^T \hat{q}_1(t)$ where $F_1(t)$ is the reaction force. Also, boundedness of $M_1(q_1), \frac{\partial M_1}{\partial q_1}(q_1), \frac{\partial^2 M_1}{\partial q_1^2}(q_1)$ and $\frac{\partial^2 M_1}{\partial q_1^2}(q_1)$ can be achieved if the master and slave configuration spaces are compact and their inertia matrices are smooth. This
condition is satisfied in many practical robots (e.g. revolute joint robots).

The proposed control law (9)-(10) requires symmetric delays (i.e. forward and backward delays are the same) and their accurate estimates. To relax this requirement which might limit applicability of the proposed scheme, we recently extend it to the case where the delays are constant but asymmetric and their estimate are inaccurate. Due to the space limitation, we don’t include it into this paper and leave it for future publication.

The key step in the proof of theorem 2 is the Parseval’s identity in (25) which we assume to be (Goldberg, 1961). As the following lemma shows, this condition can be guaranteed if the human and slave environment are passive in the sense of (13), and the Coriolis matrices in (1)-(2) are bounded w.r.t. (i.e. regardless of) $q_1, q_2$. The Parseval’s identity was also used in (Colgate and Schenkel, 1997) to ensure the energetic passivity of haptic-interfaces under zero-order-hold.

**Lemma 3.** Suppose that the human and slave environment are passive in the sense of (13) and define $\mathcal{L}_\infty$-stable impedance maps. Suppose further that $\frac{\partial M^{jk}}{\partial q_j}$ are bounded, where $M^{jk}$ are the $jk$-th and the $m$-th components of $M_i(q_i)$ and $q_i$ ($i = 1, 2$). Then, if $\dot{q}_1(0), \dot{q}_2(0), q_E(0)$ are bounded, $\dot{q}_1(t), \dot{q}_2(t) \in \mathcal{L}_2$. Thus, $q_E(t) \in \mathcal{L}_2$, and the Parseval’s identity (25) is justified.

**Proof:** If $\dot{q}_1(t), \dot{q}_2(t), q_E(t)$ are bounded, $T_i(t)$ will be bounded (from (9)-(10)), and also $F_i(t)$ will be bounded, since the human and slave environment are assumed to be $\mathcal{L}_\infty$-stable impedance map ($i = 1, 2$). Moreover, from (1)-(2) with the bounded $\frac{\partial M^{jk}}{\partial q_j}$ (i.e. the Coriolis matrices are bounded functions w.r.t $q_1, q_2$), $\dot{q}_1(t), \dot{q}_2(t)$ will also be bounded. Therefore, if $\dot{q}_1(0), \dot{q}_2(0), q_E(0)$ are bounded, $\dot{q}_1(t), \dot{q}_2(t)$ are also bounded.

With these bounded $\dot{q}_1(0), \dot{q}_2(0), q_E(0)$ ($i = 1, 2$), we can find $t > 0$ for a su–ciently large $M > 0$ s.t. $\forall t \in I := [0, t)$, $\int_0^t P(\dot{d}) < M$, where $P(t)$ is given in (14). Thus, on $I$, the Parseval’s identity (25) holds and, following the inequality (27), $\dot{q}_1(t), \dot{q}_2(t), q_E(t)$ are all bounded.

Suppose that $\dot{q}_1(t) \notin \mathcal{L}_2$ or $\dot{q}_2(t) \notin \mathcal{L}_2$. Then, because $\int_0^t P(\dot{d})$ is continuous on $I$ (since $\dot{q}_1(t), \dot{q}_2(t)$ are bounded $\forall t \in I$) and monotonically increasing $\forall t > 0$, there should exist $t_0$ and a su–ciently large $M > 0$ s.t. $0 < t_0 < t$ and $M < \int_0^{t_0} P(\dot{d}) < M$ where $M := \kappa_f(0) + V_c(0) + V_p(0) + d_1^2 + d_2^2$. However, this is not possible, because, on the interval $I$, the Parseval’s identity (25) holds, thus, (from (27), we have: for all $t \in I$,

$$0 \bullet \kappa_f(0) + V_c(0) + V_p(0) + d_1^2 + d_2^2 \int_0^t P(\dot{d}) \cdot$$

This implies that $\int_0^t P(\dot{d})$ is continuous on $I$ (i.e. $P(t)$ should be uniformly bounded by $M$ on $I$ and cannot blow up) and contradicts with the assumption that $M < \int_0^{t_0} P(\dot{d})$. Therefore, $\dot{q}_1(t), \dot{q}_2(t) \in \mathcal{L}_2$ and $q_E(t) \in \mathcal{L}_2$. Thus, following (Goldberg, 1961), Parseval’s identity (25) is valid $\forall t > 0$.

4. SIMULATION

We consider a pair of 2-links planar robots. We also model the human as PD-type position tracking controller (spring+damper). To evaluate the contact stability, we implement a lightly damped wall in the slave environment at $x = 0.35m$ reacting only along the $x$-direction. We set the gains $K_d$ and $K_p$ in (9)-(10) following the condition (12), while the dissipation gain $P_r$ is set to be 0.01$K_d$. We also impose time-delay $= 2sec$. During 0–50sec, the human operator stabilizes the slave before the wall. Then, 50–150sec, s/he pushes the slave into the wall, and 150–200sec, s/he retracts the slave from the wall.

As shown in figure 2, interaction with the wall is stable with 2sec time-delay. Master-slave position coordination is also achieved when the slave does not interact with the wall. When the slave interacts with the wall, slave force is re ected to
the human through the deformation of the spring gain $K_p$ in the control (9)-(10). A force peak in figure 2 around 58 sec occurs when the human starts moving the master. This is because of the dissipation gain $K_d$ in (9)-(10), which, according to the gain-setting condition (12), needs to be high when we want to achieve high spring gain $K_p$ for better force reflection. If we decrease $K_d$ to mitigate this force peak, we also need to decrease $K_p$ to satisfy the gain-setting condition (12), then, fidelity of force reflection would be compromised. Thus, under the gain-setting condition (12) (i.e. passivity requirement), we have bandwidth trade-off between motion agility and force reflection.

5. CONCLUSIONS

We propose a novel passive bilateral control law for nonlinear mechanical teleoperators under a constant communication time-delay. In contrast to widely-utilized scattering theory (or wave formalism) based approaches where the control and communication blocks are passified individually and position coordination is ensured only implicitly, the proposed control scheme enforces energetic passivity of the closed-loop teleoperator by passifying the combination of the communication and control blocks, and also, by explicitly communicating position signals, ensures the master-slave position coordination when there is no mismatched environment and human forces. The proposed scheme also achieves force reflection in static manipulations. Thus, performance and transparency are enhanced substantially with guaranteed interaction stability. The proposed control scheme is symmetric in the sense that the communication and control structure for the master and slave systems have the same forms. Simulation is performed to validate the properties of the proposed control scheme.

REFERENCES


