Proceedings of the 2006 American Control Conference
Minneapolis, Minnesota, USA, June 14-16, 2006

Master-Slave Synchronization with Switching Communication Through Passive Model-Based Control Design

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Abstract— **In this work we consider a master-slave robot system communicating across a bilateral communication network subject to information loss. The dynamics of the network are modelled as two binary variables. The resulting overall system is a switched system composed of master and slave with continuous dynamics, and the network which serves as switching law. The master and slave on each side of the network are provided with a model of their counterpart. We design a controller to achieve stable synchronization of the master and slave systems independent of the switching introduced by the information loss in the network. Our approach enforces passivity of the interconnected switched system using a common storage function. The passivity property guarantees stable interaction of the system with a passive environment. Such property is highly desirable in teleoperators, as well as any cooperating robotic systems.**

I. INTRODUCTION

The technological revolution that came along in the last century with the advent of wireless communication and Internet brought a breadth of innovation and a significant change in methods of communication. Interaction between systems is no longer constrained to be at the same physical location.

Thus, new applications with remotely located systems interacting with each other are emerging: assembling space structures, exploring hazardous environment, executing telesurgery, systems coordination and synchronization over a network and many others.

Networked systems increase the complexity in control analysis and design, and present new challenging problems. Although the network provides connection between remote systems, it also introduces delays and information losses that degrade the performance of the system and possibly destabilize it. Furthermore, the limited bandwidth of the network compromises our otherwise achievable control objectives. Several of this issues have been addressed in the networked-control systems community [5], [6], [11]. The problem of multiagents synchronization deals with a set of systems, called *slaves*, trying to coordinate their position and velocity with that of a *master*. If the communication among the systems is discrete, which is the case when the systems are connected across a network, and is affected by information loss, we are faced with a more complex problem i.e. synchronization of switched systems which involves continuous system dynamics and discrete communication network.

Previous work on system synchronization does not consider information loss. In [9] the problem of controlled synchronization of nonlinear systems is considered and sufficient conditions for synchronization are provided. Also in [8] self-synchronization versus controlled synchronization are described. The second case arises whenever control is used to achieve closed loop synchronization of two or more dynamical systems. In what follows we refer to synchronization as master-slave controlled synchronization. In the literature the problem of master-slave synchronization is usually referred as a unidirectional interconnection synchronization problem as mentioned in [9].

In this paper we consider the case of mutual interconnections, forward communication from master to slave, and backward communication from slave to master. A reference signal (e.g. human command) is given to the master through an external command called the primary command. Whenever the forward communication is not active then a secondary command is given to the slave as its reference signal. As the communication is affected by information loss, the synchronizing signals are not always available. We design a controller that achieves synchronization and stability for the total system overcoming the effect of the information loss. Previous work addressing the issue of information loss for discrete teleoperation has been done in [1] where they proposed a new scheme for handling lost packets while maintaining passivity. However, the approach suffered from performance degradation (position drift and loss of force feedback) since position feedback is not used there. For more details on this issue see [10].

The information loss in each channel is modelled as a binary variable $\theta_k \in \{0, 1\}$ that indicates the reception (θ_k=1) or the loss (θ_k=0) of the packet containing the signal information. Each side of the channel is provided with a model of the counterpart. The models provide an estimate of the missing states whenever communication is down in the channel. Also the models are used as an extra degree of freedom in design to achieve synchronization of master and slave and enforcing energetic passivity [12] (i.e. mechanical power as the supply rate) of the closed loop system. Energetic passivity is an important aspect of our result since it guarantees stable interaction of the system with any external passive system, such as human, environment or other robotic systems. The passivity aspect in synchronization is addressed in [14] where output synchronization of passive dynamical systems is considered when time delays are present in the

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1-4244-0210-7/06/$20.00 ©2006 IEEE

communication channel.

The novelty of our approach lies in the introduction of the master and slave models similar to the concept of model-based networked control [7]. This overcomes the problem of performance degradation when compared to other schemes, which either use buffers to replace lost packets with the previous sample or simply substitute the lost packet with a constant value. Our method enables us to achieve passivity while maintaining good synchronization performance. Furthermore our method of modeling a teleoperation system subject to information loss as a switched system extends the passivity concept into a hybrid setting. We also introduce the concept of extended passivity for connected systems, which implies passivity of the connection without necessarily requiring passivity of the individual systems. Using this concept we are equipped with a new passivity-based design approach, which allows us to design for a passive connection of subsystems which are passive on an extended space but not necessarily passive in the classical sense.

The paper is organized as follows: In Section II we describe the switched synchronization problem, providing a model for the system and the network. In Section III, we describe the concept of passivity for switched systems and provide an extension of this concept. In Section IV, we describe our main result which is the design for passivity and synchronization for the switched system. We present an example in Section V, and conclude in Section VI.

II. Problem Statement

The control architecture of remote synchronization of systems is shown in Figure 1, which illustrates the energy flow of the total system, where C_s and C_{sm} are the coordinating torques for the real slave system and slave model defined below. On the master side, a slave model provides an estimate of the slave dynamics and is used whenever communication is lost. Analogously on the slave side, a master model provides an estimate of the master dynamics for the slave. Both master and slave are provided with reference signals x_m^r, x_s^r, communicated through primary and secondary commands F_h, $\tilde{F}_h$ respectively. The slave reference signal x_s^r is used whenever x_m^r is not available. In both sides whenever communication is established, the information received is used by the master/slave. Whenever communication is lost on one side, the model provides an estimate, hence the synchronization performance depends on the amount of information loss, the accuracy of the models, and the discrepancy between master and slave reference signals. In this paper we design a controller that stabilizes the system independently of the rate of information loss, and achieves bounded error in performance under persistent losses.

The master and slave dynamics are described by the following differential equations:

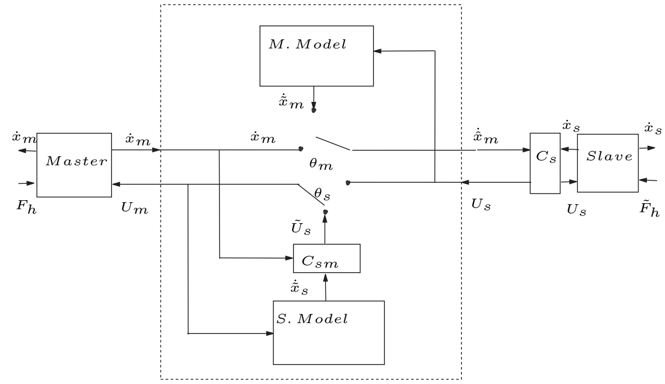

Fig. 1. Diagram of Energy Flow

$$\begin{aligned} m_1\ddot{x}_m + b_1\dot{x}_m &= -U_m(\theta_s, \theta_m) + \tau_m + F_h \\ m_2\ddot{x}_s + b_2\dot{x}_s &= U_s(\theta_m) + \tau_s + \tilde{F}_h \\ \tilde{m}_1\ddot{\tilde{x}}_m + \tilde{b}_1\dot{\tilde{x}}_m &= -U_s(\theta_m) + \tilde{\tau}_m \\ \tilde{m}_2\ddot{\tilde{x}}_s + \tilde{b}_2\dot{\tilde{x}}_s &= U_m(\theta_s, \theta_m) + \tilde{\tau}_s, \end{aligned} \tag{1}$$

where

$$\begin{aligned} U_m(\theta_m, \theta_s) &:= \theta_s \left[b_s(\dot{\hat{x}}_m - \dot{x}_s) + k_s(\hat{x}_m - x_s) \right] + \\ &\quad (1-\theta_s) \left[\tilde{b}_s(\dot{x}_m - \dot{\tilde{x}}_s) + \tilde{k}_s(x_m - \tilde{x}_s) \right], \end{aligned} \tag{2}$$

$$\begin{aligned} U_s(\theta_m) &:= \theta_m \left[b_s(\dot{x}_m - \dot{x}_s) + k_s(x_m - x_s) \right] + \\ &\quad (1-\theta_m) \left[b_s(\dot{\tilde{x}}_m - \dot{x}_s) + k_s(\tilde{x}_m - x_s) \right] \end{aligned} \tag{3}$$

and, with an abuse of notation (i.e. $\tilde{\dot{x}}_m = \dot{\tilde{x}}_m$), $\hat{\star}_m := \theta_m \star_m + (1-\theta_m)\tilde{\star}_m$, x_m, $\tilde{x}_m$, x_s, $\tilde{x}_s \in \mathbb{R}$ are the master, master model, slave and slave model state respectively. The constants b_i, $\tilde{b}_i$, m_i, $\tilde{m}_i$, $i = 1, 2$; $b_s, \tilde{b}_s$, $k_s, \tilde{k}_s$ are scalar and real-valued. U_s is the slave coordinating force, and U_m is coordinating force exiting from the network. The $\tau_\star$, $\tilde{\tau}_\star$, $\star \in \{m, s\}$ are the additional control to be designed later to achieve master-slave synchronization and passivity of the closed-loop master-slave system. Also we define the slave model coordinating force

$$\tilde{U}_s := \left[\tilde{b}_s(\dot{x}_m - \dot{\tilde{x}}_s) + \tilde{k}_s(x_m - \tilde{x}_s) \right]. \tag{4}$$

In between the two systems the communication network is modelled as two binary variables $\theta_m(k)$, $\theta_s(k) \in \{0, 1\}$ that indicate the reception or the loss of the packets containing the signal information. F_h represents a primary external command, for example the human force, which gives a reference signal to the master. $\tilde{F}_h$ is a secondary command to the slave.

We define a general piecewise (right or left) continuous switching signal $\theta : t \to \{1, 2, 3, 4\}$ as $\theta = \theta_m\theta_s + 2(1-\theta_m)\theta_s + 3\theta_m(1-\theta_s) + 4(1-\theta_m)(1-\theta_s)$. The system assumes four possible configurations defined by the family of subsystems $F = \{f_\theta,\, \theta \in \Theta = \{1, 2, 3, 4\}\}$. Consider the two-port system composed of master and slave models and the two channel communication network as in Figure 1 with input $u = [\dot{x}_m \; -U_s]'$ and output $y = [U_m \; \dot{\tilde{x}}_m]'$, then the net power input for this block is given by $u^T y = U_m \dot{x}_m - U_s \dot{\tilde{x}}_m$. Different values of θ_m and θ_s define each subsystem in the family F. For the power of the network block we have the following four values

(a) When both the channels are connected
$\theta_m = \theta_s = 1,\, \theta = 1 \Rightarrow u^T y = 0$

(b) In the case of backward connection only
$\theta_m = 0,\, \theta_s = 1,\, \theta = 2 \Rightarrow$
$u^T y = \left[b_s(\dot{\tilde{x}}_m - \dot{x}_s) + k_s(\tilde{x}_m - x_s)\right](\dot{x}_m - \dot{\tilde{x}}_m)$.

(c) If only the forward channel is connected
$\theta_m = 1,\, \theta_s = 0,\, \theta = 3 \Rightarrow$
$u^T y = ([\tilde{b}_s(\dot{x}_m - \dot{\tilde{x}}_s) + \tilde{k}_s(x_m - \tilde{x}_s)] - [b_s(\dot{x}_m - \dot{x}_s) + k_s(x_m - x_s)])\dot{x}_m$

(d) In the case of no connection in between the systems
$\theta_m = \theta_s = 0,\, \theta = 4 \Rightarrow$
$u^T y = [\tilde{b}_s(\dot{x}_m - \dot{\tilde{x}}_s) + \tilde{k}_s(x_m - \tilde{x}_s)]\dot{x}_m - [b_s(\dot{\tilde{x}}_m - \dot{x}_s) + k_s(\tilde{x}_m - x_s)]\dot{\tilde{x}}_m$

With this setting we have a switched system comprised of a family of four different subsystems where the switching signal $\theta \in \{1, 2, 3, 4\}$ is defined by the two binary variables θ_m, θ_s.

III. Hybrid Passivity

It is well-known [3] that the interconnection of p-port passive systems is passive. Hence we can decompose the passivity design problem for the total system, into passivity design for the sub-blocks. In fact passivity of the overall system is guaranteed if the three sub-blocks, master and slave and the network block including the models (i.e. master, slave, and network block in Figure 1) are individually rendered to be passive. We prove that if we include the external blocks dynamics in the *extended passivity* analysis, defined in what follows, passivity is achieved for the overall system. In order to use the master and slave dynamics to achieve passivity of the internal block, we need to slightly extend the classical concept of passivity. The next definition and the following lemma provide the extension we need.

Consider a p-port switched system

$$\begin{aligned} \dot{x} &= f_{\theta(t)}(x) + g_{\theta(t)}(x)u \\ y &= h_{\theta(t)}(x) \end{aligned} \tag{5}$$

where $x \in X \subseteq \mathbb{R}^n$, $u \in U \subset \mathbb{R}^p$ and $y \in \mathbb{R}^p$ are the system state, input and output respectively. U is the set of admissible inputs. f_i, g_i, h_i, $i \in S = \{1, \ldots, s\}$ are smooth vector fields with $f_i(0) = 0$, $g_i(0) = 0$, $h_i(0) = 0$. The switching rule θ is a piecewise constant function such that $\theta : [0, \infty) \to \Theta$. The corresponding system for $\theta(t) = i \in \Theta$ is called the i^{th} subsystem. We say that the i^{th} subsystem is active whenever $\theta(t) = i$.

Definition 1: The $p-$port switched system (5) is said to be **passive**, in $(0, \infty) \times \mathbb{R}^n$ if there exists a nonnegative function $V \in C^1$, such that $V : \mathbb{R} \times \mathbb{R}^n \to R^+$, $V(\theta(t), 0) = 0$, $\forall t$, and $\forall u \in U$, $x_0 \in X$, $t \in (0, \infty)$, $\dot{V}(\theta(t), x(t)) \leq u^T y$

$\diamond$

Definition 2: Consider a $p-$port switched system (5) connected to another $p-$port switched system described by

$$\begin{aligned} \dot{z} &= \bar{f}_{\theta(t)}(z) + \bar{g}_{\theta(t)}(z)(-y),\; z \in \mathbb{R}^{n_z} \\ u &= \bar{h}_{\theta(t)}(z) \end{aligned} \tag{6}$$

Where the functions $\bar{f}, \bar{g}, \bar{h}$ have the same properties of f, g, h. The systems (5) and (6) share the common switching law $\theta(t)$. Then we is say that system (5) is **passive on the extended space** $\mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^{n_z}$, if there exists a nonnegative function $V(\theta, x, z) \in C^1$, defined on the augmented state space $\mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^{n_z}$ with $V(\theta(t), 0, 0) = 0$, $\forall t$, such that $\forall u \in U$, $x_0 \in X, z_0 \in Z$, $t \in (0, \infty)$

$$\dot{V}(\theta(t), x(t), z(t)) \leq u^T y \tag{7}$$

Moreover if the condition (7) is satisfied in absolute value, i.e. $\dot{V}(\theta(t), x(t), z(t)) \leq |u^T y|$, then both systems (5) and (6) are passive on the extended space $\mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^{n_z}$.

$\diamond$

Before we state sufficient conditions for passivity in the extended space for a switched system, we recall from switched system theory that stability of a switched system [4] can be analyzed through the use of a common Lyapunov function for all the subsystems, or using several Lyapunov functions satisfying proper conditions at the switching boundaries. Analogously, those concepts can be extended to analyze hybrid passivity, in which single or multiple storage functions are used [13], [2]. In the same vein, the following theorems provide sufficient conditions for passivity and passivity with extended space of the switched system (5), using a common storage function for the family of subsystems.

Theorem 1: [2] Consider a p-port switched system (5), with switching signal θ, input u and output y. Assume that $\forall i$ f_i, g_i are locally Lipschitz continuous in x and $u(t)$ is a measurable function in t. Moreover assume that the state does not jump at the switching instants, and therefore the solution $x(.)$ is continuous everywhere. If all the subsystems in the family $F = \{f_\theta,\, \theta \in \Theta\}$ share a *common storage function*, i.e. $V : \mathbb{R}^n \Rightarrow \mathbb{R}$, $V \in C^1$, $\frac{\partial V}{\partial x} f_\theta(x) \leq u^T y$, $\forall x$, $\forall \theta \in \Theta$, then the switched system is passive for all switching sequences.

$\diamond$

A similar result can be proven for the case of extended space.

Theorem 2: Consider the p-port switched system (5), satisfying the same assumptions as in Theorem 1, connected to system (6). If all the subsystems in the family $F = \{f_\theta,\, \theta \in \Theta\}$ share a *common storage function*, defined on the extended space $\mathbb{R}^{n_z} \times \mathbb{R}^n$ i.e. $V : \mathbb{R}^{n_z} \times \mathbb{R}^n \to \mathbb{R}$, $V \in C^1$

$$\frac{\partial V}{\partial x} f_\theta(x) + \frac{\partial V}{\partial z} \bar{f}_\theta(z) \leq u^T y,\; \forall x \in \mathbb{R}^n,\; \forall z \in \mathbb{R}^{n_z},\; \forall \theta \in \Theta$$

then the switched system is passive on the extended space $\mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^{n_z}$ for all switching sequences.

$\diamond$

The next lemma shows that the interconnection of systems which are passive on the compatible extended space results in another passive system.

Lemma 1: Consider a connection of three two-port switched systems depicted in Figure 2. Let $x_1 \in \mathbb{R}^{n_1}$, $x_2 \in$

$\mathbb{R}^{n_2}$, $x_3 \in \mathbb{R}^{n_3}$ be the first, second and third systems states respectively, and let $\theta \in \{1, \dots, s\}$ be the switching sequence which defines a family of s systems.

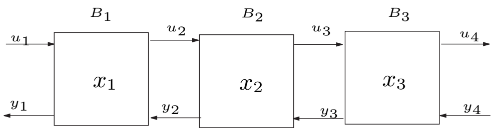

Fig. 2. Connection of three two-ports switched systems with common switching sequence θ

Assume there exists positive definite storage functions $V_i : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \times \mathbb{R}^{n_3}$, $i = 1, 2, 3$, $V_i > 0$, $V_i(0, 0, 0) = 0$, such that $\forall \theta$

$$
\begin{aligned}
u_1^T y_1 - u_2^T y_2 &\geq \dot{V}_1(x_1, x_2, x_3) \\
u_2^T y_2 - u_3^T y_3 &\geq \dot{V}_2(x_1, x_2, x_3) \\
u_3^T y_3 - u_4^T y_4 &\geq \dot{V}_3(x_1, x_2, x_3)
\end{aligned}
$$

i.e. the sub-blocks are individually passive with extended space $\mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \times \mathbb{R}^{n_3}$ and storage functions V_i, $i = 1, 2, 3$. Then the switched system resulting from the connection of these three blocks is passive for all switching sequences with $V = V_1 + V_2 + V_3$ as its total storage function.

Proof. The result simply follows from Theorem 1 with storage function $V = V_1 + V_2 + V_3$, in fact we have $V = \sum_{i=1}^{3} V_i > 0$, $u_1^T y_1 - u_4^T y_4 \geq \dot{V}(x_1, x_2, x_3)$, $\forall \theta \in \Theta$.

$\diamond$

IV. Passivity-Based Control Design for Synchronization of Switched System

In order to enforce passivity of the total interconnection in Figure 1, we can first enforce passivity on the extended space of the network-models block, where the state space of the models is extended to include the master and slave state spaces. Next we need to enforce passivity for the master, and slave blocks, respectively, with state space extended to the state-space of the master and slave models.

We introduce the following notation for compactness

$$
\begin{aligned}
e_{ms} &= U_m(1,1) = U_s(1) \\
e_{\tilde{m}s} &= U_m(0,1) = U_s(0) \\
e_{m\tilde{s}} &= U_m(*,0)
\end{aligned}
$$

Theorem 3: Consider the synchronization system described by (1). Then the two-port system composed of network and models is passive with extended space $\mathbb{R}^4$, if the following hybrid controllers are applied

$$
\begin{aligned}
\tau_m &= -(1 - b_1)\dot{x}_m + (1 - \theta_m)\theta_s(-2e_{\tilde{m}s}) \\
&\quad +(1 - \theta_m)(1 - \theta_s)(e_{m\tilde{s}}) \\
\tilde{\tau}_m &= -(1 - \tilde{b}_1)\dot{\tilde{x}}_m + \theta_m\theta_s(-2e_{ms}) \\
&\quad +\theta_m(1 - \theta_s)(-2e_{ms}) \\
\tau_s &= -(1 - b_1)\dot{x}_s + (1 - \theta_m)(1 - \theta_s)(-e_{\tilde{m}s}) \\
\tilde{\tau}_s &= -(1 - \tilde{b}_1)\dot{\tilde{x}}_s + \theta_m\theta_s(2e_{ms})
\end{aligned}
\tag{8}
$$

Where $b_s, \tilde{b}_s, k_s, \tilde{k}_s$ in the coordinating torques are such that $b_s = k_s$, $\tilde{b}_s = \tilde{k}_s$ and $b_s \geq \tilde{b}_s$.

Proof. *Consider the switched system described by (1). Then the network-models block is passive (i.e. satisfies definition 2), if there exists a common storage function defined on the extended state space $\mathbb{R}^4$ for which all the four subsystems defined by the switching rule θ satisfy the passivity condition*

$$\dot{V} \leq u^T y. \tag{9}$$

Consider the storage function $V : \mathbb{R}^4 \rightarrow \mathbb{R}, V = V(\tilde{x}_m, \tilde{x}_s, x_m, x_s)$

$$
\begin{aligned}
V &= \frac{1}{2}(2\dot{x}_m^2 + \dot{x}_s^2 + \dot{\tilde{x}}_m^2 + \dot{\tilde{x}}_s^2 \\
&\quad + \frac{2.5}{b_s}e_{ms}^2 + \frac{0.75}{b_s}e_{\tilde{m}s}^2 + \frac{2.75}{\tilde{b}_s}e_{m\tilde{s}}^2)
\end{aligned}
\tag{10}
$$

we have to verify that each subsystem in the family $F\{f_i,\ i = 1, \dots, 4\}$ satisfy the passivity condition (9) with respect to this storage function. If we consider the derivative of the storage function along the system trajectory, we have

$$
\begin{aligned}
\dot{V} &= 2\dot{x}_m(\ddot{x}_m) + \dot{x}_s(\ddot{x}_s) + \dot{\tilde{x}}_m(\ddot{\tilde{x}}_m) + \dot{\tilde{x}}_s(\ddot{\tilde{x}}_s) + \\
&\quad \frac{2.5}{b_s}e_{ms}(\dot{e}_{ms}) + \frac{0.75}{b_s}e_{\tilde{m}s}(\dot{e}_{\tilde{m}s}) + \frac{2.75}{\tilde{b}_s}e_{m\tilde{s}}(\dot{e}_{m\tilde{s}})
\end{aligned}
\tag{11}
$$

We proceed by proving that the passivity condition (9) is satisfied for the four cases described in section II.

(a) *First in the full connection case we have $\theta_M = \theta_S = 1$ and $u^T y = 0$. The system dynamics have the form*

$$
\begin{aligned}
\ddot{x}_m &= -(\dot{x}_m + e_{ms}) + F_h \\
\ddot{x}_s &= -(\dot{x}_s - e_{ms}) \\
\ddot{\tilde{x}}_m &= -(\dot{\tilde{x}}_m - e_{ms}) \\
\ddot{\tilde{x}}_s &= -(\dot{\tilde{x}}_s + e_{ms})
\end{aligned}
$$

and therefore

$$
\begin{aligned}
\dot{e}_{m\tilde{s}} &= \dot{e}_{\tilde{m}s} = 0 \\
\dot{e}_{ms} &= -2b_s e_{ms}
\end{aligned}
\tag{12}
$$

Substituting the dynamics in (11) we obtain

$$
\begin{aligned}
\dot{V} &= 2\dot{x}_m(-\dot{x}_m - e_{ms}) + \dot{x}_s(-\dot{x}_s + e_{ms}) + \\
&\quad \dot{\tilde{x}}_m(-\dot{\tilde{x}}_m + e_{ms}) + \dot{\tilde{x}}_s(-\dot{\tilde{x}}_s - e_{ms}) \\
&\quad -5e_{ms}^2 \leq 0
\end{aligned}
\tag{13}
$$

This implies that the subsystem (a) with full connection is passive with storage function (10).

(b) *When only the slave to master channel is connected we have $\theta_M = 0$, $\theta_S = 1$, $u^T y = e_{\tilde{m}s}(\dot{x}_m - \dot{\tilde{x}}_m)$ and the following dynamics for the system*

$$
\begin{aligned}
\ddot{x}_m &= -(\dot{x}_m - e_{\tilde{m}s}) \\
\ddot{x}_s &= -(\dot{x}_s - e_{\tilde{m}s}) + \tilde{F}_h \\
\ddot{\tilde{x}}_m &= -(\dot{\tilde{x}}_m + e_{\tilde{m}s}) \\
\ddot{\tilde{x}}_s &= -(\dot{\tilde{x}}_s - e_{\tilde{m}s})
\end{aligned}
\tag{14}
$$

analogously to the first case we have

$$\begin{aligned} \dot{e}_{ms} &= \dot{e}_{m\tilde{s}} = 0 \\ \dot{e}_{\tilde{m}s} &= -2b_s e_{\tilde{m}s} \end{aligned} \tag{15}$$

Substituting the dynamics in (11) we obtain

$$\begin{aligned} \dot{V} = & \, 2\dot{x}_m(-\dot{x}_m + e_{\tilde{m}s}) + \dot{x}_s(-\dot{x}_s + e_{\tilde{m}s}) + \\ & \dot{\tilde{x}}_m(-\dot{\tilde{x}}_m - e_{\tilde{m}s}) + \dot{\tilde{x}}_s(-\dot{\tilde{x}}_s + e_{\tilde{m}s}) - \\ & 1.5e_{\tilde{m}s}^2 \leq e_{\tilde{m}s}\dot{x}_m - e_{\tilde{m}s}\dot{\tilde{x}}_m \end{aligned} \tag{16}$$

This implies the subsystem (b) with backward connection is passive with storage function (10).

(c) *When only the connection between master to slave is active we have $\theta_M = 1$, $\theta_S = 0$, $u^T y = (e_{m\tilde{s}} - e_{ms})\dot{x}_m$ and the following dynamics for the system*

$$\begin{aligned} \ddot{x}_m &= -(\dot{x}_m + e_{m\tilde{s}}) + F_h \\ \ddot{x}_s &= -(\dot{x}_s - e_{ms}) \\ \ddot{\tilde{x}}_m &= -(\dot{\tilde{x}}_m - e_{ms}) \\ \ddot{\tilde{x}}_s &= -(\dot{\tilde{x}}_s - e_{m\tilde{s}}) \end{aligned} \tag{17}$$

analogously to the first case we have

$$\begin{aligned} \dot{e}_{m\tilde{s}} &= -2\tilde{b}_s e_{m\tilde{s}} \\ \dot{e}_{ms} &= -(\tilde{b}_s e_{m\tilde{s}} + b_s e_{ms}) \\ \dot{e}_{\tilde{m}s} &= 0 \end{aligned} \tag{18}$$

Substituting the dynamics in (11) we obtain

$$\begin{aligned} \dot{V} = & \, 2\dot{x}_m(-\dot{x}_m - \tilde{U}_s) + \dot{x}_s(-\dot{x}_s + e_{ms}) + \\ & \dot{\tilde{x}}_m(-\dot{\tilde{x}}_m + e_{ms}) + \dot{\tilde{x}}_s(-\dot{\tilde{x}}_s + e_{m\tilde{s}}) \\ & -2.5e_{ms}^2 - 2.5\frac{\tilde{b}_s}{b_s}\tilde{U}_s e_{ms} - 5.5\tilde{U}_s^2 \\ & \leq -e_{ms}\dot{x}_m + e_{m\tilde{s}}\dot{x}_m \end{aligned} \tag{19}$$

This given that the conditions on b_s given in the theorem, are satisfied, implies the subsystem (c) with forward connection is passive with storage function (10).

(d) *Finally in the case of both channel disconnected we have $\theta_M = 0$, $\theta_S = 0$, $u^T y = e_{\tilde{m}s}(\dot{x}_m - \dot{\tilde{x}}_m) + (\tilde{U}_s - e_{\tilde{m}s})\dot{x}_m$ with the following dynamics for the system*

$$\begin{aligned} \ddot{x}_m &= -\dot{x}_m + F_h \\ \ddot{x}_s &= -\dot{x}_s + \tilde{F}_h \\ \ddot{\tilde{x}}_m &= -(\dot{\tilde{x}}_m + e_{\tilde{m}s}) \\ \ddot{\tilde{x}}_s &= -(\dot{\tilde{x}}_s - e_{m\tilde{s}}) \end{aligned} \tag{20}$$

analogously to the first case we have

$$\begin{aligned} \dot{e}_{m\tilde{s}} &= -\tilde{b}_s e_{m\tilde{s}} \\ \dot{e}_{ms} &= 0 \\ \dot{e}_{\tilde{m}s} &= -b_s e_{\tilde{m}s} \end{aligned} \tag{21}$$

Substituting the dynamics in (11) we obtain

$$\begin{aligned} \dot{V} = & \, 2\dot{x}_m(-\dot{x}_m) + \dot{x}_s(-\dot{x}_s) + \dot{\tilde{x}}_m(-\dot{\tilde{x}}_m - e_{\tilde{m}s}) \\ & +\dot{\tilde{x}}_s(-\dot{\tilde{x}}_s + e_{m\tilde{s}}) - 0.75e_{\tilde{m}s}^2 - 2.75e_{m\tilde{s}}^2 \leq \\ & -e_{\tilde{m}s}\dot{\tilde{x}}_m + e_{m\tilde{s}}\dot{x}_m \end{aligned} \tag{22}$$

This implies the subsystem (d) with no connection is passive with storage function (10).

◇

Next consider the master and the slave blocks in Figure 1. The controllers (8) guarantee passivity with extended space of the master and slave blocks.

Corollary 1: Consider the master and slave in (1) with hybrid controllers (8). Then the master and the slave are passive on the extended space that includes the dynamics of the models.

Proof: The result can be easily proven with the storage functions $V_m = \frac{1}{2}(\dot{x}_m^2 + e_{\tilde{m}s}^2 + e_{ms}^2 + e_{m\tilde{s}}^2)$, for the master, and $V_s = \frac{1}{2}(\dot{x}_m^2 + \dot{\tilde{x}}_m^2 + e_{\tilde{m}s}^2 + e_{ms}^2 + e_{m\tilde{s}}^2)$, for the slave.

◇

Theorem 1 gives us a set of hybrid controllers for master slave and the relative models, which guarantee passivity with extended space of the combination of the network and the models. Also Corollary 1 proves that such controller ensures extended passivity of individual master and slave. Then from Lemma 1 we have that total interconnection in Figure 1 (i.e. total closed-loop system) is passive.

The choice of a common storage function which includes master-slave errors dynamics, guarantee not only passivity for the internal block (network plus models), but also good steady state performance in term of low tracking errors. In particular whenever partial or full communication is available the tracking error converges to zero. Due to the fast response of the resulting subsystems, we have passivity even in the case of fast switching. A bound on the tracking error can be obtained by analyzing the dynamics in the worse case, i.e. when the system are disconnected on both channels. In this case the tracking error can be bounded by the errors between the reference signals.

V. Example

We simulate the synchronization system using the following values for the systems parameters $m_1 = 1$, $b_1 = 1.2$, $m_2 = 2$, $b_2 = 1.12$, $\tilde{m}_1 = 1.5$, $\tilde{b}_1 = 1.7$, $\tilde{m}_2 = 2.5$, $\tilde{b}_2 = 1.9$, $k_s = b_s = \frac{13}{6}$, $\tilde{k}_s = \tilde{b}_s = \frac{4}{3}$. The sampling time for the master, slave and respective models is $T_s = 10^{-3}sec$, and the network sampling time is $T_n = 1sec$. The reference for the master is $x_r = 3.5sin(2\pi 0.1t)$ and for the master model $\tilde{x}_r = 3sin(2\pi 0.2t)$. The simulation results are depicted in Figure 3. The first diagram of the figure shows the two sequences θ_m, θ_s. The second plot shows the master and slave position coordination. Notice that when at least one of the channels is active, i.e. $\theta_m = 1$ ($\theta_s = 0$) or $\theta_s = 1$ ($\theta_m = 0$) the master and slave synchronize. The peak in the errors occur when both the forward and backward communications become inactive such that the master and slave follow different references. As long as the reference signals have bounded discrepancy the errors between master and slave remain bounded. In a second simulation we considered the same setting as in the previous example. The reference input to the master is given through

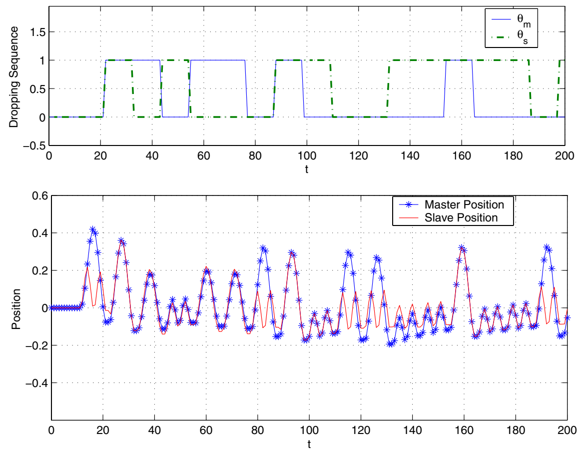

Fig. 3. Switching sequence θ generated by the sequences θ_m, θ_s, master and slave trajectories.

an external force F_h and on the slave side an environmental force F_e is considered. The human and slave environment are modeled as spring damper systems with their spring set positions being $10m$ and $0m$, respectively. In Figure 4, are depicted the switching sequences on top, and the forces on the bottom. In the case of full communication the forces F_h, F_e match and have a nonzero value. When only backward communication is available the forces are both zero, since the slaves does not see the master, (i.e. uses the master model) and hence does not exhort any force on the environment. Thus the master perceive zero error between slave and master model position. When only forward communication is available, there is a drift in the forces since the slave sees the master and hence apply a force on the environment, on the other side the master is only connected with the slave model, hence it perceives zero reaction force hence the error in position between master and slave model is zero, as well as the master force.

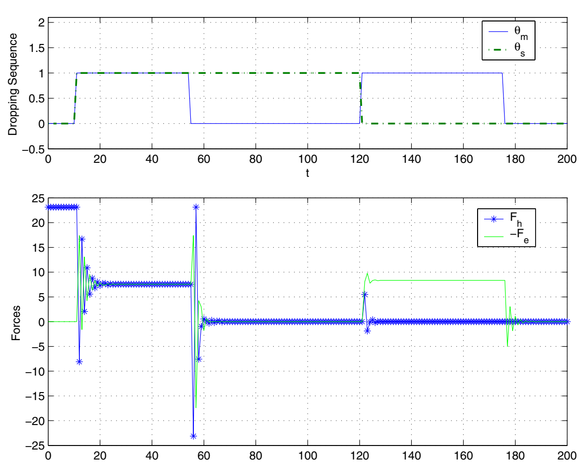

Fig. 4. Switching sequence θ generated by the sequences θ_m, θ_s, Forces at the master and at the slave.

VI. Conclusion

We considered the synchronization problem of the master-slave system across a communication network. The network is subject to information loss on both forward and backward channels. This model results in a switched system. Through a passivity based approach we designed a hybrid controller which guarantees stable interconnection and bounded synchronization error via Lyapunov-type analysis. Passivity of the total system is guaranteed independently of the switching sequence, i.e. even for very fast switching the performance might decrease but passivity and stability are preserved. Synchronization performance depends on the amount of information received, model accuracy and gap between master and slave reference signals. We are currently working on extending our result to the case of one master and multiple slaves systems with possibly delays included.

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