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Stable Flocking of Multiple Inertial Agents on Balanced Graphs

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***Abstract*— We consider the flocking of multiple agents which have significant inertias and evolve on a balanced information graph. We first show that flocking algorithms that neglect agents' inertial effect can cause unstable group behavior. To incorporate this inertial effect, we use the passive decomposition, which decomposes the closed-loop group dynamics into two decoupled systems: a shape system representing the internal group formation, and a locked system describing the motion of the center-of-mass. Then, analyzing the locked and shape systems separately with the help of graph theory, we propose a provably-stable flocking control law, which ensures that the internal group formation is exponentially stabilized to a desired shape, while all the agents' velocities converge to the centroid velocity that is also shown to be time-invariant. This result still holds for slowly-switching balanced information graphs. Simulation is performed to validate the theory.**

***Index Terms*— multiagent flocking, distributed coordination, information graph, decomposition**

I. Introduction

The multiagent distributed coordination problem has received much attention from many researchers (see [1] for a collection of such efforts). One of the key research questions is how to design a local action of possibly simple agents (with limited computing power and sensing capability) such that, collectively, a certain desired pattern/behaviour can emerge. Once we have an answer to this question, we would be able not only to realize many powerful engineering applications (e.g. mobile sensor networks and distributed robotic surveillance/rescue [2], [3], [4]), but also to understand many fascinating phenomenon in nature (e.g. schooling of fishes [5], human collective behaviour [6]).

Compared to conventional control problems, the unique challenge of the multiagent distributed coordination is how to analyze the information topology among the agents, i.e., which agents sense/are sensed by which agents. This is important, because it determines how the local action propagates throughout the group. However, at the same time, only certain information topologies are feasible, especially when the number of agents is large.

To analyze this information topology, graph theory has been used in many works (e.g. [7]-[15]), where the information topology is represented by its information graph. Among them, the majority assumes that the evolution of each agent can be described well enough by its kinematics, i.e. single integrator dynamics (e.g. [7]-[10]). However, in many important applications (e.g. robots, spacecraft), it is not generally possible to directly control the velocity, as most of the actuators (e.g. torque motor, gas jet) can affect only the acceleration through the agents' inertias. Moreover, as shown in this paper, for a certain (directed) information topology, this agents' inertial effect can even cause unstable group behaviors. Therefore, it is necessary to incorporate this inertial effect (i.e. double integrator dynamics) into the multiagent distributed coordination control design.

In this paper, we present a novel distributed coordination framework for multiple agents with significant inertial effect and evolving on a balanced information graph [7] (i.e. in-degree = out-degree for all agents). This class of balanced graphs includes two practically important classes of information topology: undirected graphs (i.e. inter-agent communication is two-way) and cyclic graphs [16]. We consider the particular problem of flocking [11], [12], where all the agents are desired to move with a common velocity, while keeping a certain desired internal group formation.

To deal with the agent's inertial effect, we use the passive decomposition [17], [18], which decomposes the closed-loop group dynamics into two decoupled systems: a shape system representing the internal group formation shape, and a locked system describing the dynamics of the centroid (i.e. center-of-mass). Then, by analyzing the locked and shape systems separately with some results of graph theory, we propose a provably-stable flocking control law, which ensures that the internal group formation is exponentially stabilized to a desired shape, while all the agents' velocities converge to the centroid velocity that is also shown to be time-invariant. Using the dwell-time concept [19], we can also show that this result still holds for slowly-switching balanced information graphs.

To our knowledge, in other distributed coordination schemes, the agents' inertial effect has been considered only for the cases where either the information graph is undirected so that the closed-loop group dynamics becomes usual mass-spring-damper dynamics (e.g. [11], [12], [20], [21]), or all the agents' dynamics are identical so that the effects of the agent's dynamics and the information topology can be separated from each other by using Kronecker algebra [13], [14], [15]. In contrast, our proposed framework is applicable even when the agents' inertias are all different and the information graph is a general balanced graph. In this sense,

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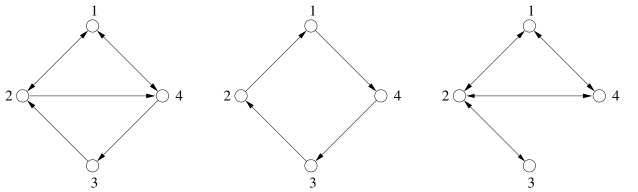

Fig. 1. Some examples of strongly connected balanced information graphs. Balanced graphs include cyclic (second) and undirected (third) graphs.

this present work may be thought of as an extension of the results in [7] (i.e. coordination of kinematic agents on a balanced graph) and [11], [12] (i.e. flocking of inertial agents on an undirected graph).

The rest of the paper is organized as follows. Section II provides some preliminary graph theory and an example highlighting the importance of the agents' inertial effect. Then, in Section III, using the passive decomposition, the closed-loop group dynamics is decomposed into the locked and shape systems. In Section IV, the novel flocking algorithm for multiple inertial agents on a balanced graph is presented and its properties are detailed. In Section V, simulations are performed, and Section VI contains some concluding remarks.

II. Information Graph and Motivating Example

Consider n-agents. Then, the information topology among them can be represented by their (weighted and directed) information graph $G := \{\mathcal{V}, \mathcal{E}, \mathcal{W}\}$, where $\mathcal{V} := \{v_1, ..., v_n\}$ is the set of nodes (i.e. agents), $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges (i.e. ordered pairs of the nodes), and $\mathcal{W} : \mathcal{E} \rightarrow \Re^+$ is a map assigning a (positive) weight to each edge s.t. if $e_{ij} := (v_i, v_j) \in \mathcal{E}$, $\mathcal{W}(e_{ij}) = w_{ij}$, $w_{ij} > 0$. See Figure 1 for some examples. For notational convenience, we exclude the self-joining edges from $\mathcal{E}$, i.e. $e_{ii} \notin \mathcal{E}$, $\forall i \in \{1, ..., n\}$. Here, $e_{ij} = (v_i, v_j) \in \mathcal{E}$ (i.e. v_i and v_j are the head and tail of the edge e_{ij}, respectively) would imply that the information flows from v_j to v_i. This would happen, if the agent i tries to follow the state of the agent j. The weighting w_{ij} can be useful if each information flow has non-uniform reliability (e.g. different signal-to-noise ratio). For more details on the graph theory, refer to [7] and references therein.

Now, following [7], [9], let us consider the kinematics-based flocking model. Then, the closed-loop kinematics of the agent i can be given by

$$\dot{x}_i = \sum_{j \in \mathcal{N}_i} -k w_{ij}(x_i - x_j) \tag{1}$$

where x_i is the position of the agent i, $w_{ij} > 0$ is the weight assigned to e_{ij}, $k > 0$ is the control gain, and $\mathcal{N}_i$ is the information neighbor of the agent i defined by

$$\mathcal{N}_i := \{j \mid e_{ij} = (v_i, v_j) \in \mathcal{E}\} \tag{2}$$

i.e. the set of all the tails of the agent i. Here, for simplicity, we assume that the desired internal group formation is given

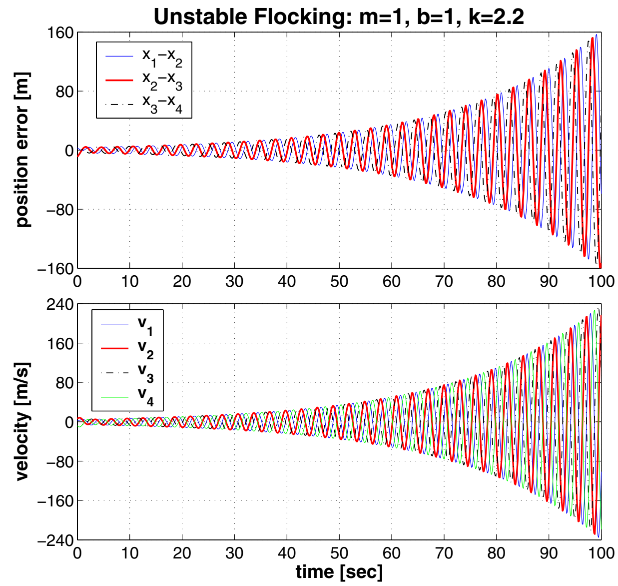

Fig. 2. Unstable flocking on the cyclic graph: $(m_i, w_{ij}, b, k) = (1, 1, 1, 2.2)$.

by $x_1 = x_2 = ... = x_n$, although the one with constant offsets (i.e. $x_i = x_j + a_{ij}$ with a constant a_{ij}) can also be easily incorporated.

Then, by stacking up the individual kinematics (1), the closed-loop group kinematics can be written as

$$\dot{x} = -kLx \tag{3}$$

where $x = [x_1, x_2, ..., x_n]^T \in \Re^n$, and the matrix $L \in \Re^{n \times n}$ is so called the Laplacian matrix of the information graph G defined s.t.

$$L_{ij} = \begin{cases} -w_{ij} & \text{if} \quad i \neq j \;\text{ and }\; (v_i, v_j) \in \mathcal{E} \\ 0 & \text{if} \quad i \neq j \;\text{ and }\; (v_i, v_j) \notin \mathcal{E} \\ \sum_{k \in \mathcal{N}_i} w_{ik} & \text{if} \quad i = j \end{cases} \tag{4}$$

where L_{ij} is the ij-th component of L.

For node i, we define the in-degree $in_i(G)$ and the out-degree $out_i(G)$ s.t. $in_i(G) := L_{ii}$ $(= -\sum_{j=1, j\neq i}^{n} L_{ij}$ from (4)) and $out_i(G) := -\sum_{j=1, j \neq i}^{n} L_{ji}$. Note that, if the graph is non-weighted (i.e. $w_{ij} = 1$), $in_i(G)$ and $out_i(G)$ are the numbers of incoming and outgoing edges of v_i, respectively.

This graph Laplacian matrix L has the following properties [7], [13]: 1) its eigenvalues have non-negative real-part (from Geršgorin's disk theorem [22]); 2) $u := [1, 1, .., 1]^T \in \Re^n$ is an eigenvector with zero eigenvalue, i.e. $Lu = 0$, since $\sum_{j=1}^{n} L_{ij} = 0$ from (4); and 3) if G is strongly connected, this zero eigenvalue is simple. Because all the eigenvalues of the Laplacian matrix L have non-negative real-parts, the closed-loop group kinematics (3) is stable $\forall k \geq 0$.

In many practical applications (e.g. robots, spacecraft), it is generally possible to directly control the acceleration rather than the velocity. Thus, we incorporate the agents' inertias into the flocking model. Similar to (1), let us consider the following closed-loop dynamics of the agent i with its inertia $m_i > 0$

$$m_i \ddot{x}_i = \sum_{j\in\mathcal{N}_i} -bw_{ij}(\dot{x}_i - \dot{x}_j) - kw_{ij}(x_i - x_j) \tag{5}$$

where $b, k > 0$ are the damping and stiffness gains. Then, similar to (3), the closed-loop group dynamics can be written as

$$M\ddot{x} + bL\dot{x} + kLx = 0 \tag{6}$$

where $M := diag[m_1, m_2, ..., m_n] \in \Re^{n\times n}$.

If we apply this dynamics-based flocking model (6) for the four agents on the cyclic graph (second of Figure 1) with $(m_i, w_{ij}, b, k) = (1, 1, 1, 2.2)$, we see that the group behavior is unstable as shown in Figure 2. Note that, if G is undirected as in [11], [12], the dynamics (6) becomes a usual linear time-invariant mass-spring-damper system with a symmetric and positive-semidefinite L, thus, stable $\forall b, k \geq 0$. This example clearly shows the importance of the interaction between the agents' inertias and information topology, and the necessity for a framework geared toward agents with non-negligible inertias and evolving on general directed information graphs.

III. DECOMPOSITION

In this section, using the passive decomposition [17], [18], we decompose the closed-loop group dynamics (6) into two systems: a shape system describing the internal group formation, and a locked system describing the motion of the center-of-mass.

Following [17], we define the following coordinate transformation:

$$z := \underbrace{\begin{bmatrix} \frac{m_1}{\sum_{i=1}^n m_i} & \frac{m_2}{\sum_{i=1}^n m_i} & \frac{m_3}{\sum_{i=1}^n m_i} & \cdots & \frac{m_n}{\sum_{i=1}^n m_i} \\ 1 & -1 & 0 & \ldots & 0 \\ 0 & 1 & -1 & \ldots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ldots & \ldots & 1 & -1 \end{bmatrix}}_{=:S\in\Re^{n\quad n}} x \tag{7}$$

where $z := [z_1, z_2, ..., z_n]^T \in \Re^n$ is the transformed coordinate, and S is the (full-rank) transformation matrix. Let us define $z_e := [z_2, z_3, ..., z_n]^T \in \Re^{(n-1)}$ so that $z = [z_1, z_e^T]^T$. Then, from (7), we can show that z_e describes the internal group formation shape as it is given by

$$z_e = [x_1 - x_2, x_2 - x_3, ..., x_{n-1} - x_n]^T \tag{8}$$

and z_1 abstracts the overall group maneuver, as

$$z_1 = \frac{1}{\sum_{i=1}^n m_i}(m_1 x_1 + m_2 x_2 + ... + m_n x_n) \tag{9}$$

i.e. the position of the *centroid*. We designate z_1 and z_e as the configurations of the locked and shape systems, respectively, whose dynamics will be defined below.

Using (7), we can rewrite the closed-loop group dynamics (6) with respect to z such that

$$S^{-T}MS^{-1}\ddot{z} + bS^{-T}LS^{-1}\dot{z} + kS^{-T}LS^{-1}z = 0 \tag{10}$$

where the inverse of S in (7) is given by [17], [18]

$$S^{-1} = \begin{bmatrix} 1 & \phi_2 & \phi_3 & \ldots & \phi_n \\ 1 & \phi_2 - 1 & \phi_3 & \ldots & \phi_n \\ 1 & \phi_2 - 1 & \phi_3 - 1 & \ldots & \phi_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \phi_2 - 1 & \phi_3 - 1 & \ldots & \phi_n - 1 \end{bmatrix} \tag{11}$$

with $\phi_i := \sum_{k=i}^n m_k / \sum_{j=1}^n m_j$.

From (11) and the property of the Laplacian matrix L that $\sum_{j=1}^n L_{ij} = 0 \ \forall i \in \{1, ..., n\}$, we can show that 1) the inertia matrix M in (10) is block-diagonalized s.t.

$$S^{-T}MS^{-1} =: diag[m_L, \bar{M}] \tag{12}$$

where $m_L := \sum_{i=1}^n m_i > 0$ and $\bar{M} \in \Re^{(n-1)\times(n-1)}$ is a symmetric and positive-definite matrix; and 2) the transformed information graph in (10) has the following structure:

$$S^{-T}LS^{-1} = \begin{bmatrix} 0 & \bar{D}^T \\ 0_{(n-1)\times 1} & \bar{L} \end{bmatrix} \tag{13}$$

where the j-th component of $\bar{D} \in \Re^{(n-1)}$ is given by

$$\bar{D}_j := -\sum_{k=j+1}^n (L_{1k} + L_{2k} + ... + L_{nk}) \tag{14}$$

$j \in \{1, 2, ..., n-1\}$, and the ij-th component of the matrix $\bar{L} \in \Re^{(n-1)\times(n-1)}$ is given by

$$\bar{L}_{ij} = \phi_{i+1}\bar{D}_j + \sum_{p=i+1}^n \sum_{q=j+1}^n L_{pq} \tag{15}$$

$i, j \in \{1, 2, ..., n-1\}$. The following Proposition is a direct consequence of (7)-(15).

Proposition 1 *The closed-loop group dynamics* (6) *can be decomposed as*

$$m_L \ddot{z}_1 + b\bar{D}^T \dot{z}_e + k\bar{D}^T z_e = 0 \tag{16}$$
$$\bar{M}\ddot{z}_e + b\bar{L}\dot{z}_e + k\bar{L}z_e = 0 \tag{17}$$

where $m_L = \sum_{i=1}^n m_i > 0$*, and* $\bar{D}$ *and* $\bar{L}$ *are defined in* (14)-(15).

We call the dynamics of z_e (17) *shape system*, which describes the internal group formation shape as given by (8). We also call the dynamics of z_1 in (16) *locked system*, which represents the centroid motion. Notice from (16)-(17) that the shape system is completely decoupled from the locked system, while the locked system is coupled with the shape system via the coupling term $\bar{D}$.

IV. Flocking on Balanced Graphs

In this section, we consider the flocking problem of multiple inertial agents on strongly connected balanced graphs [7], i.e., any two nodes are connected (i.e. path exists between them) and each node has the same in-degree and out-degree. See Figure 1 for examples. We will utilize following properties of balanced graphs.

Proposition 2 1) *The following are equivalent:*

a) *The information graph G is balanced.*
b) *All the column sums of L are zeros, i.e. $\sum_{j=1}^{n} L_{ji} = 0$, $\forall i \in \{1, ..., n\}$.*
c) *The coupling term $\bar{D}$ in* (14) *vanishes.*
d) *The matrix $\bar{L}$ in* (15) *becomes inertia-independent with its ij-th component given by*

$$\bar{L}_{ij} = \sum_{p=i+1}^{n} \sum_{q=j+1}^{n} L_{pq}. \tag{18}$$

2) *If the graph G is balanced and strongly connected, $\bar{L}_{sym} := \frac{1}{2}(\bar{L} + \bar{L}^T)$ is positive-definite.*

Proof: 1-b) The graph G is balanced, if and only if $in_i(G) = L_{ii} = out_i(G) = -\sum_{j=1, j\neq i}^{n} L_{ji}$, $\forall i \in \{1, ..., n\}$, where we use the expressions for $in_i(G)$ and $out_i(G)$ in Section II. This condition is equivalent to $\sum_{j=1}^{n} L_{ji} = in_i(G) - out_i(G) = 0$, $\forall i \in \{1, ..., n\}$.
1-c) Following the above item 1-b, if the graph G is balanced, all the column sums are zeros. Thus, from (14), $\bar{D}_j = 0$, thus, $\bar{D} = 0$. For the necessity, suppose that $\bar{D} = 0$. Then, $\bar{D}_{n-1} = -\sum_{j=1}^{n} L_{jn} = 0$, i.e. the n-th column sum of L should be zero. However, since $\bar{D}_{n-2} = \bar{D}_{n-1} - \sum_{j=1}^{n} L_{j(n-1)}$, if $\bar{D} = 0$, the $(n-1)$-th column sum of L should also be zero. By continuing this process up to $\bar{D}_1$, we have $\sum_{j=1}^{n} L_{ji} = 0$ for $i = 2, ..., n$ (i.e. all the column sums of L except the first column are zeros). However, since all the row sums of L are zeros (see Section II), $\sum_{i=1}^{n} \sum_{j=1}^{n} L_{ij} = 0$ (i.e. sum of all elements of L is zero). Therefore, the first column sum of L should also be zero. Then, following the above item 1-b, G is necessarily balanced.
1-d) This is a direct consequence of the expression (15), the above item 1-c, and the fact that $\phi_i > 0$, $\forall i \in \{2, 3, ..., n\}$.
2) From (13) with $\bar{D} = 0$ (from item 1-c of this Proposition), we can show that $S^{-T} L_{sym} S^{-1} = diag[0, \bar{L}_{sym}]$, where $L_{sym} := \frac{1}{2}(L + L^T)$. Since S^{-1} is full-rank, this is a similarity transform, thus, preserves the locations of eigenvalues. Also, if G is balanced and strongly connected, L_{sym} has only one eigenvalue at zero with all the others being strictly positive real [7, theorem 7]. Therefore, $\bar{L}_{sym}$ is positive-definite. ∎

Therefore, if the graph G is balanced, the centroid dynamics (i.e. locked system in (16)) will be decoupled from the internal group formation (i.e. shape system (17)), and both the locked and shape systems can be analyzed separately. Note that if G is not balanced, such a complete decoupling cannot be ensured.

The difficulty in analyzing the shape system (17) is due to the fact that $\bar{L}$ is generally asymmetric. This difficulty, however, can be overcome for the balanced graphs, by using the fact that $\bar{L}_{sym}$ is positive-definite (item 2 of Proposition 2). We now present the main result.

Theorem 1 *Consider the closed-loop group dynamics* (6) *and their locked and shape dynamics* (16)-(17).

1) *For arbitrary initial conditions and gains (b, k), the centroid velocity $\dot{z}_1(t)$ is invariant, i.e.*

$$\dot{z}_1(t) = \frac{\sum_{i=1}^{n} m_i \dot{x}_i(t)}{\sum_{i=1}^{n} m_i} = \dot{z}_1(0) \quad \forall t \geq 0 \tag{19}$$

if and only if the information graph G is balanced.

2) *Suppose that the graph G is balanced and strongly connected. Suppose further that we set the gains $b, k > 0$ s.t.*

$$b^2 \bar{L}_{sym} - k\bar{M} \succ 0 \tag{20}$$

where $A \succ 0$ implies that $A \in \Re^{n\times n}$ is positive-definite. Then, $(\dot{z}_e(t), z_e(t)) \to 0$ exponentially, and

$$\dot{x}_i(t) \to \dot{z}_1(t) = \dot{z}_1(0) = \frac{\sum_{i=1}^{n} m_i \dot{x}_i(0)}{\sum_{i=1}^{n} m_i} \tag{21}$$

$\forall i = 1, ..., n$, i.e. the internal group formation converges to the desired shape, while all the agents' velocities converge to the invariant centroid velocity.

Proof: 1) Suppose that the graph G is balanced. Then, from Proposition 2, $\bar{D} = 0$ and the locked system dynamics (16) becomes

$$m_L \ddot{z}_1 = 0 \tag{22}$$

where z_1 is defined in (9) and $m_L = \sum_{i=1}^{n} m_i$. Since $\ddot{z}_1(t) = 0$ $\forall t \geq 0$ from (22), we have $\dot{z}_1(t) = \dot{z}_1(0)$, $\forall t \geq 0$. For the necessity, suppose that $\dot{z}_1(t) = \dot{z}_1(0)$, $\forall t \geq 0$. Then, from (16), we have $\int_0^t \bar{D}(b\dot{z}_e(\theta) + kz_e(\theta))d\theta = 0$ $\forall t \geq 0$, for arbitrary $z_e(t), \dot{z}_e(t)$. This requires that $\bar{D} = 0$. Following item 1 of Proposition 2, this implies that the graph G is necessarily balanced.
2) Let us consider the shape system dynamics (17), and decompose $\bar{L}$ s.t. $\bar{L} = \bar{L}_{sym} + \bar{L}_{skew}$, where $\bar{L}_{skew} = \frac{1}{2}(\bar{L} - \bar{L}^T)$. From item 2 of Proposition 2, $\bar{L}_{sym}$ is positive-definite, since the graph G is strongly connected and balanced.

Let us define a Lyapunov function candidate

$$V := \frac{1}{2} \begin{pmatrix} \dot{z}_e \\ z_e \end{pmatrix}^T P \begin{pmatrix} \dot{z}_e \\ z_e \end{pmatrix} \tag{23}$$

with

$$P := \begin{bmatrix} \bar{M} & \epsilon\bar{M} \\ \epsilon\bar{M} & (k + \epsilon b)\bar{L}_{sym} \end{bmatrix} \in \Re^{2(n-1)\times 2(n-1)} \tag{24}$$

where $\epsilon > 0$ is to be designed below s.t. P is positive-definite. Then, using the shape dynamics (17), and the fact that $v^T\bar{L}w = w^T\bar{L}^Tv = \frac{1}{2}v^T\bar{L}w + \frac{1}{2}w^T\bar{L}^Tv \ \forall v, w \in \Re^n$, we can show that

$$\frac{dV}{dt} = -\begin{pmatrix}\dot{z}_e \\ z_e\end{pmatrix}^T Q \begin{pmatrix}\dot{z}_e \\ z_e\end{pmatrix} \tag{25}$$

where $Q \in \Re^{2(n-1)\times 2(n-1)}$ is given by

$$Q := \begin{bmatrix} b\bar{L}_{sym} - \epsilon\bar{M} & \frac{1}{2}(k-\epsilon b)\bar{L}_{skew} \\ -\frac{1}{2}(k-\epsilon b)\bar{L}_{skew} & \epsilon k\bar{L}_{sym} \end{bmatrix}. \tag{26}$$

Let us choose $\epsilon = k/b$ to cancel out the off-diagonal terms of Q in (26). Then, Q will be positive-definite if and only if $b^2\bar{L}_{sym} - k\bar{M} \succ 0$. Also, following [22, p.472], P in (24) with $\epsilon = k/b$ will be positive-definite if and only if $2b^2\bar{L}_{sym} \succ k\bar{M}$. Thus, if $b^2\bar{L}_{sym} - k\bar{M}$ is set to be positive-definite as stated in this Theorem, both P and Q will be positive-definite, and $(\dot{z}_e(t), z_e(t)) \to 0$ exponentially. From (8), this implies that $(\dot{x}_i(t) - \dot{x}_j(t), x_i(t) - x_j(t)) \to 0$ exponentially $\forall i, j \in \{1, 2, ..., n\}$. With $\dot{x}_i(t) - \dot{x}_j(t) \to 0$, from (19), we also have $\dot{z}_1(t) - \dot{x}_i(t) \to 0$, $\forall i = 1, ..., n$. Moreover, since the graph G is balanced, following item 1 of this Theorem, $\dot{z}_1(t) = \dot{z}_1(0)$, $\forall t \geq 0$. Therefore, we have $\dot{x}_i(t) \to \dot{z}_1(t) = \dot{z}_1(0)$. ■

Item 1 of Theorem 1 still holds for switching information graphs if they are all balanced, since, in that case, the dynamics (22) is ensured regardless of the switching. On the other hand, since the shape dynamics (17) under the condition (20) is linear and exponentially stable, following [19, Lemma 2], item 2 of Theorem 1 also still holds for switching balanced information graphs, if the condition (20) is satisfied for each switched information graph and the switching is slow enough in the sense that the interval between any consecutive switchings is no smaller than a dwell-time $\tau_o > 0$. This dwell-time can be estimated by analyzing the dynamics (17) with all possible switching information graphs. For more details, refer to [19].

The condition (20) says that the internal group formation will be stabilized if the damping b and the "good" part of the information topology (i.e. $\bar{L}_{sym}$) are strong enough, compared to the stiffness gain k and the shape inertia $\bar{M}$. Note from item 1-d of Proposition 2 that, if the information graph G is balanced, $\bar{L}_{sym}$ can be directly computed from the graph structure G without knowing agents' inertias, since the expression (18) is inertia-independent. In contrast, the computation of $\bar{M}$ requires only the inertia structure, as its expression in (12) is independent on the information topology.

The condition (20) is simple to use but possibly conservative, since 1) it assumes a specific structure of the Lyapunov function (23)-(24); and 2) it does not utilize any structural information of $\bar{L}_{skew}$, as its design aims to get rid of the effects of $\bar{L}_{skew}$ by cancelling out the off-diagonal terms of Q in (26). This conservatism can be partially reduced by directly enforce $P \succ 0$ and $Q \succ 0$. Note that these two conditions are linear matrix inequalities (LMIs) w.r.t. ϵ. Thus, by solving these LMIs w.r.t. ϵ using available commercial software, we can easily check if a pair of b, k will guarantee stable flocking or can possibly induce unstable behaviors.

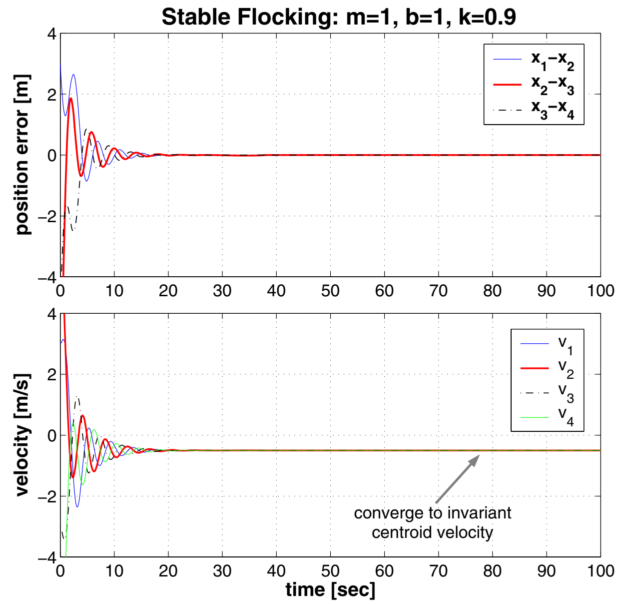

Fig. 3. Stable flocking on the cyclic graph with the gain-setting condition (20): $(m_i, w_{ij}, b, k) = (1, 1, 1, 0.9)$

Remark 1 *Following Theorem 1, if the graph G is strongly connected and balanced, all the agents will reach the position and velocity agreement:* $x_i(t) - x_j(t) \to 0$ *and* $\dot{x}_i(t) - \dot{x}_j(t) \to 0$. *Moreover, if all the agents' inertias are the same (i.e.* $m_i = m_j \ \forall i, j$ *in* (5)*), we will achieve velocity average-consensus [7], i.e.,* $\dot{x}_i(t) \to \frac{1}{n}(\dot{x}_1(0) + \dot{x}_2(0) + ...\dot{x}_n(0))$.

V. Simulation

We apply the results of Theorem 1 to the example of Section II (i.e. four agents on the cyclic graph). As shown in Figure 3, with the condition (20), the group behavior now becomes stable. Also, the desired internal group formation is achieved (top of Figure 3), and individual agents' velocities converge to the invariant centroid velocity (bottom of Figure 3). Since we set all the agents' inertias to be same, as stated in remark 1, the velocity average-consensus is also achieved.

In Figure 4, we use the less conservative LMI conditions stated in the paragraph before remark 1. The desired internal group shape and the velocity average-consensus are still achieved. However, as the design becomes less conservative, the group behavior becomes more oscillatory than that in Figure 3. This LMI-based condition becomes infeasible when $k \geq 2$ with $m_i = w_{ij} = b = 1$. This well matches with the fact that the group behavior becomes unstable for k larger than 2 with $(m_i, w_{ij}, b) = (1, 1, 1)$ (e.g. Figure 2).

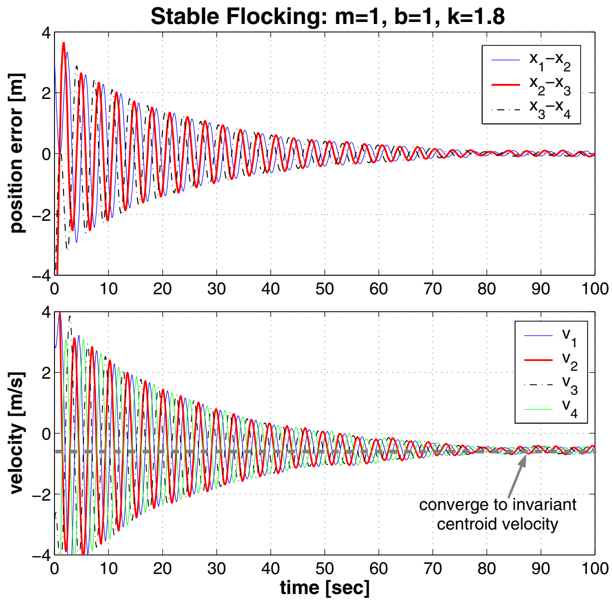

Fig. 4. Stable flocking on the cyclic graph with the LMI conditions $P \succ 0$, $Q \succ 0$: $(m_i, w_{ij}, b, k) = (1, 1, 1, 1.8)$

We also impose some noise and model uncertainty into the simulations. As our stability is exponential (item 2 of Theorem 1), the group behavior with such perturbations turn out to be close enough to the one without them, as long as their magnitudes are small enough. Therefore, the results with such perturbations are omitted here.

VI. Conclusions

In this paper, we consider the multiagent flocking problem. We first show that, for a certain directed information graph, the group behavior can go unstable, if the agents' inertial effect is neglected. Then, we propose a novel provably-stable flocking framework for multiple agents which have significant inertial effects and evolve on balanced information graphs. By relying on the passive decomposition and the graph theory, the proposed framework ensures that the internal group formation is exponentially stabilized to a desired shape, while the velocities of all the agents converge to the (time-invariant) centroid velocity.

There are several research directions we will pursue in future. The gain-setting condition (20) (or the LMI condition) is only sufficient for stability. Since the matrices in the decomposed dynamics possess specific structures and properties (e.g. (18)), using matrix theory, we may be able to exploit such properties/structure to reduce the conservatism. Relying on the dwell-time concept [19], we show that the proposed framework is also applicable for slowly-switching balanced graphs. However, it is not yet clear how each system parameter affects the dwell-time, as they are all mingled together in the system dynamics (17). Of particular interest is to quantify the effect of the information topology on the dwell-time. We would also like to extend this framework to the case where information topology and agents' motion are coupled with each other.

References

[1] V. Kumar, N. Leonard, and A. S. Morse, editors. *Cooperative Control*. Springer, 2004. Lecture Notes in Control and Information Sciences 309.

[2] B. Sinopoli, C. Sharp, L. Schenato, S. Schaffert, and S. S. Sastry. Distributed control applications within sensor networks. *Proceedings of the IEEE*, 91(8):1235–1246, 2003.

[3] P. Orgen, E. Fiorelli, and N. E. Leonard. Cooperative control of mobile sensor networks: Adaptive gradient climbing in a distributed environment. *IEEE Transactions on Automatic Control*, 49(8):1292–1302, 2004.

[4] V. Kumar, D. Rus, and S. Singh. Robot and sensor networks for first responders. *IEEE Pervasive Computing*, 3(4):24–33, 2004.

[5] T. Vicsek. A question of scale. *Nature*, 411(May 24):421, 2001.

[6] D. J. Low. Following the crowd. *Nature*, 407(Sep. 28):465–466, 2000.

[7] R. Olfati-Saber and R. M. Murray. Consensus problems in networks of agents with switching topology and time-delays. *IEEE Transactions on Automatic Control*, 49(9):1520–1533, 2004.

[8] A. Jadbabaie, J. Lin, and A. S. Morse. Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Transactions on Automatic Control*, 48(6):988–1001, 2003.

[9] Z. Lin, M. E. Broucke, and B. A. Francis. Local control strategies for groups of mobile autonomous agents. *IEEE Transactions on Automatic Control*, 49(4):622–629, 2004.

[10] W. Ren and R. Beard. Consensus seeking in multiagent systems under dynamically changing interaction topologies. *IEEE Transactions on Automatic Control*, 50(5):655–661, 2005.

[11] H. G. Tanner, A. Jadbabai, and G. J. Pappas. Stable flocking of mobile agents, part i: fixed topology. In *Proceedings of the 42nd IEEE Conference on Decision and Control*, pages 2010–2015, 2003.

[12] H. G. Tanner, A. Jadbabai, and G. J. Pappas. Stable flocking of mobile agents, part ii: dynamic topology. In *Proceedings of the 42nd IEEE Conference on Decision and Control*, pages 2016–2021, 2003.

[13] J. A. Fax and R. M. Murray. Information flow and cooperative control of vehicle formations. *IEEE Transactions on Automatic Control*, 49(9):1465–1476, 2003.

[14] G. Lafferriere, J. Caughman, and A. Williams. Graph theoretic methods in the stability of vehicle formations. In *Proceeding of the American Control Conference*, pages 3729–3734, 2004.

[15] G. Lafferriere, A. Williams, J. Caughman, and J. J. P. Veerman. Decentralized control of vehicle formation. *Systems and Control Letters*, 54:899–910, 2005.

[16] J. A. Marshall, M. E. Broucke, and B. A. Francis. Formations of vehicles in cyclic pursuit. *IEEE Transactions on Automatic Control*, 49(11):1963–1974, 2004.

[17] D. J. Lee and P. Y. Li. Formation and maneuver control for multiple spacecrafts. In *Proceeding of American Control Conference*, volume 1, pages 278–283, 2003.

[18] Dongjun Lee. *Passive Decomposition and Control of Interactive Mechanical Systems under Coordination Requirements*. Doctoral Dissertation, University of Minnesota, 2004. Also available at http://decision.csl.uiuc.edu/∼d-lee/Dissertation.pdf.

[19] A. S. Morse. Supervisory control of families of linear set-point controllers, part i: exact matching. *IEEE Transactions on Automatic Control*, 41(10):1413–1431, 1996.

[20] W. Ren and R. W. Beard. Decentralized scheme for spacecraft formation flying via virtual structure approach. *AIAA Journal of Guidance, Control, and Dynamics*, 27(1):73–82, 2004.

[21] D. J. Lee and P. Y. Li. Decomposition and control of multiple spacecraft formation flying. *International Journal of Control*. Submitted, also available at http://decision.csl.uiuc.edu/∼d-lee/LeeLiIJC05.pdf.

[22] R. A. Horn and C. R. Johnson. *Matrix analysis*. Cambridge University Press, Cambridge, UK, 1985.